STUDY OF DIFFERENTIAL ATTITUDE ENGINE ON-TIME CONTROL TO OBTAIN EXTREMELY LOW LIMIT CYCLE RATES

FINAL REPORT

REPORT NO. 2309-950001

APRIL 1966

Prepared under Contract NAS 8-20212

by Textron's Bell Aerosystems Company
Buffalo, N.Y.

for

National Aeronautics and Space Administration
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FOREWORD

This final report summarizes and documents the study effort conducted by Bell Aerosystems Company, under NASA Contract NAS 8-20212, during the period from 29 June 1965 to 30 April 1966.

The NASA Technical Supervisor for this contract was Mr. David Schultz, Astrionics Laboratory, Marshall Space Flight Center.

The study was performed by the Integrated Systems Engineering Department, Bell Aerosystems Co., Buffalo, N. Y. The chief contributors to the study were:

Donald C. Neil, Technical Director
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ABSTRACT

A novel technique for obtaining small net impulse bits is described and evaluated. This Differential On-Time Control (DOTC) technique is based on the concurrent firing of two attitude control thrusters, whose moment contributions in at least one axis tend to cancel, for slightly different lengths of time. Several DOTC concepts are developed for the control of a "base line" space vehicle with roll/yaw thrusters arranged such that each one produces a relatively large control acceleration in roll and a much smaller control acceleration in yaw. The various DOTC concepts are evaluated over a range of key parameters centered about the baseline vehicle and mission characteristics, and compared with Linear Signal Mixing and conventional control concepts on the basis of propellant consumption rates, thruster actuation rates, and logic complexity. DOTC shows significant performance advantages over the best of other concepts under essentially undisturbed conditions, and provides comparable performance in the presence of the largest environmental disturbances likely to be encountered in orbit. DOTC logic is more complex, and places greater demands on sensor and thruster performance. However, all component requirements appear to be within the current state-of-the-art.

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LIST OF SYMBOLS

Symbol	Definition	Units
I	Moment of inertia	$kg - m^2$
ℓ	Pitch and yaw axis moment arm	m
r	Roll axis moment arm	m
A, B, O, P, S	Roll axis DOTC variables	
C, D	Yaw axis DOTC variables	
A_R, B_R	Roll rate DOTC variables	
C_R, D_R	Yaw rate DOTC variables	
T	Thruster and thrust level designation	Newtons
a, b	Attitude rate feedback gains	deg/(deg/sec)
V	Thruster command signal	volts
t_{min}, t_p	Minimum equivalent square wave pulse width	sec
τR	Turn on delay equivalent to thruster decay dynamics	sec
Δt	Equivalent square wave differential on-time	sec
ϕ	Attitude position angle	deg
$\dot{\phi}$	Attitude rate	deg/sec
$\ddot{\phi}$	Attitude acceleration	deg/sec^2
ϕ_{DB}	Attitude dead band value	deg

SUBSCRIPTS

x, y, z	Refers to roll, pitch, yaw axes, respectively	
1,2,...11,12	Thruster designation numbers	

ABBREVIATIONS

YP	Yaw priority
RP	Roll priority
DBB	Roll control by differential back-to back thruster firing
DYT	Roll control by differential yaw thruster shut off
DOTC	Differential on time control
LSM	Linear signal mixing
UCC	Uncoupled controls

1.0 INTRODUCTION

This is a final report summarizing results of a Study of Differential On-Time Control Techniques conducted under NASA Contract NAS8-20212. The primary objective of this study was to investigate the feasibility of using differential on-time control (DOTC) techniques to achieve long period limit cycle attitude control of space vehicles utilizing reaction jet engines.

DOTC refers, in the simplest case, to the technique of obtaining small and repeatable impulse bits by terminating, in a time differential manner, the operation of thrusters fired in a directly opposing, or back-to-back, sense. More sophisticated DOTC concepts may involve a differential termination of thrusters initially fired to obtain control about one axis of a vehicle, so as to provide a precise moment impulse bit about another vehicle axis. Because of the long slender nature of most boosters and many space vehicles, the roll axis requires the well controlled, small impulse bits. DOTC techniques of this type can, with practically achievable differential timing, attain impulse bit control limited only by thruster "tail off" uncertainty.

Although very small bits and the associated very long undisturbed limit cycle periods can be achieved in this way, DOTC techniques possess the following adverse characteristics.

(1) In back-to-back firing, DOTC utilizes more propellant per unit impulse than for conventional rocket firing.

(2) The basic DOTC concept is not generally favorable during appreciable disturbance situations, where small impulse bits may be a disadvantage.

(3) Logic for employment of DOTC concepts is usually somewhat more complex than for conventional logics.

Though these potential disadvantages are present, missions, vehicles and thruster arrangements exist where the DOTC concept pays off. One objective of the study is to indicate those situations where DOTC concepts can be employed advantageously.

Emphasis in the DOTC studies was placed on one and two degree of freedom undisturbed limit cycle attitude control of vehicles with long, slender shapes where the inertia in roll is very much less than for pitch and yaw. It is frequently convenient for this type of vehicle to utilize the same thrusters for roll control as are employed for either pitch or yaw. However, with the thrusters sized to balance pitch or yaw disturbances and provide adequate response to maneuver commands, it is difficult to design for suitably long limit cycle periods in roll without special techniques, such as DOTC.

Primary performance criterion for the evaluation of DOTC concepts was the number of engine firings and propellant consumption during given periods of time. Although the primary advantage of DOTC occurs in undisturbed operation, the ability of the system to function acceptably under various types of disturbance conditions was also investigated.

This DOTC study has been centered about a representative relatively long slender space vehicle described in Section 2.0. However, the results have been made applicable to vehicles with a wide range of physical characteristics operating in a wide array of possible missions through establishment of key parameters, nondimensionalized where possible.

The DOTC techniques which have evolved from this study have been evaluated relative to a linear signal mixing (LSM) control technique described in Section 3.0 and in Reference 1. The principal performance criteria, propellant requirements and number of firings, are also evaluated for the DOTC systems relative to conventional control techniques.

The report is organized into four primary sections to follow. Section 2.0 describes program scope and constraints. This is followed by Section 3.0 which presents descriptions of DOTC logic concepts which have been formulated. For reference purposes the LSM control logic is also defined. These are evaluated analytically to the extent that this is practical in Section 4.0. Section 5.0 describes analog and digital simulation studies which have been accomplished under this program. These simulation programs were developed both as development and evaluation tools. Evaluation results of DOTC versus the LSM concept for attitude control are presented in that section. A final section presents the conclusions and recommendations.

2.0 DEFINITION OF MISSION, VEHICLE AND ENVIRONMENTAL PARAMETERS

The performance of DOTC relative to more conventional systems depends upon the particular mission and vehicle it is applied to. In the subject study it was desired to provide results applicable to a broad range of vehicles and missions. The approach taken has been to establish a baseline vehicle and mission, to define significant parameters (nondimensional to the extent practical) and to specify ranges of these parameters which cover the desired range of vehicles and missions. In the following sections, relevant characteristics of the mission, including requirements on attitude control accuracy, maneuver requirements, etc, are defined first. Vehicle characteristics important to this study, including all relevant aspects of their subsystems, are then defined.

2.1 MISSION CHARACTERISTICS

The characteristics of missions which largely influence the design of space vehicle attitude control systems, include:

The vehicle flight situation. Orbit injection, earth orbit, lunar or interplanetary flight and/or reentry flight phases. Of principal importance here are the altitudes (which largely determines, for a given vehicle, the aerodynamic and gravity gradient influences) the mission phase times and orbit ellipticity.

Nominal vehicle attitudes during the mission phases. This aspect is of particular importance relative to the influence of aerodynamic, gravity gradient and solar pressure torques. For some missions, the vehicle will be stabilized relative to the earth, for others, relative to the sun or stars, and perhaps in other cases, relative to another vehicle in space.

Vehicle attitude accuracy requirements during all phases of the mission. The mission objectives will usually specify these requirements. In inactive phases of some missions, it may be necessary to control attitude only within fairly wide tolerances of ± 10 to 20°. In other phases, where inertial objects are being tracked, where reconnaissance data are obtained, or where antennas for data transmission must be properly oriented, the requirements on attitude excursions and/or attitude rates are much more severe.

Magnitude and time duration of short term disturbances resulting from staging operations or translation rocket firings to modify the vehicle velocity vector. Staging disturbances can usually be approximated by an angular impulse imparted to the vehicle which must be countered by the attitude control system. Translation rocket firings will impart moments due to misalignment unless the rocket is gimbaled. If gimbaled, small moments are introduced during shutdown which must be controlled by the attitude control system.

Vehicle attitude maneuvering requirements. Maneuvers are required for a wide variety of purposes, such as: to orient sensors to align with inertial objects, to maintain an earth stable platform while in elliptic orbit and to align translation thrust vector prior to firing.

Primary emphasis in this study was on the essentially undisturbed limit cycle mode of control system operation. Of secondary importance was operation in the presence of long term low level disturbances, such as are produced by the influence of aerodynamic gravity gradient or solar pressure effects. Consistent with this emphasis, a baseline mission involving essentially undisturbed space flight, has been established. This "mission" is descriptive of the space flight of a vehicle which is: (1) out of the atmosphere where aerodynamic torques are negligible, or the vehicle is so designed and/or in such an attitude region that aerodynamic forces do not produce aerodynamic torques; (2) in a region of space flight where solar radiation does not exist in appreciable magnitude and/or where the vehicle is so designed or in such an attitude as not to experience solar radiation torques; and (3) the gravity field is of inconsequential magnitude or the vehicle physical characteristics and/or attitude relative to the gravity field are such as not to result in gravity gradient torques. Though the baseline mission did not include the influence of these disturbance torques, the various DOTC concepts which were developed for the baseline mission were evaluated also in an environment where these torques did exist.

In the specification of the mission, the time duration of the various mission segments was treated parametrically with time running from very short periods on the order of fractions of an orbit, to indefinitely long time periods. Similarly attitude control accuracy requirements were treated parametrically.

2.2 VEHICLE CHARACTERISTICS

Vehicle characteristics of importance to the design of an attitude control system (ACS) for space vehicles, and to the relative merits of conventional and DOTC systems, can be conveniently grouped according to vehicle physical characteristics, ACS thruster characteristics, ACS sensor characteristics, and ACS logic. The latter characteristic, ACS logic, is largely the subject and variable of investigation in the subject study, and is treated separately in Section III. The former characteristics are those which are assumed specified, though not quantitatively. The approach used in their definition is similar to that used in defining the mission. Baseline characteristics are specified and the significant parameters are parametrically treated.

2.2.1 Vehicle Physical Characteristics

Included here are the inertial and geometric parameters. Of importance are both the nominal characteristics and the deviation of these parameters due to various non-ideal influences.

2.2.1.1 Nominal Physical Characteristics

The nominal space vehicle defined for this study including a nominal six engine thruster configuration is illustrated in Figure 1a. In Figure 1b the same basic vehicle with an alternate eight engine thruster arrangement is shown. The principal emphasis has been on the six engine arrangement: all results, however, are equally applicable to the eight engine design.

Figure 2 lists the physical characteristics of the baseline vehicle. The quantitative values given are representative of a nominal design for which DOTC concepts were investigated. In most cases the sensitivity of system performance to fairly wide variations in the important vehicle parameters was also investigated. Vehicle physical parameters for which the influence of variations are of chief interest are: thrust (T), inertia (I), and angular acceleration $(T\ell)/(I)$. The importance of the latter parameter can be seen from examination of the following set of simplified equations of motion for the vehicle of Figure 1a.

Pitch Axis:

$$\ddot{\phi}_y = \frac{57.3}{I_y} \ell (T_{12} - T_6) + \ddot{\phi}_{y_D}$$

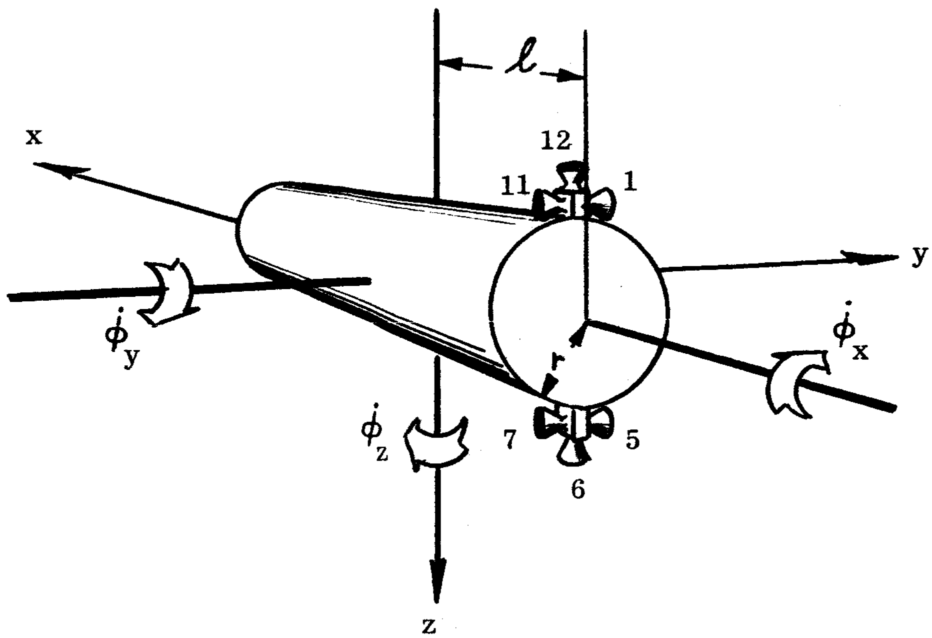
Roll Axis:

$$\ddot{\phi}_x = \frac{57.3}{I_x} r (-T_1 + T_5 - T_7 + T_{11}) + \ddot{\phi}_{x_D}$$

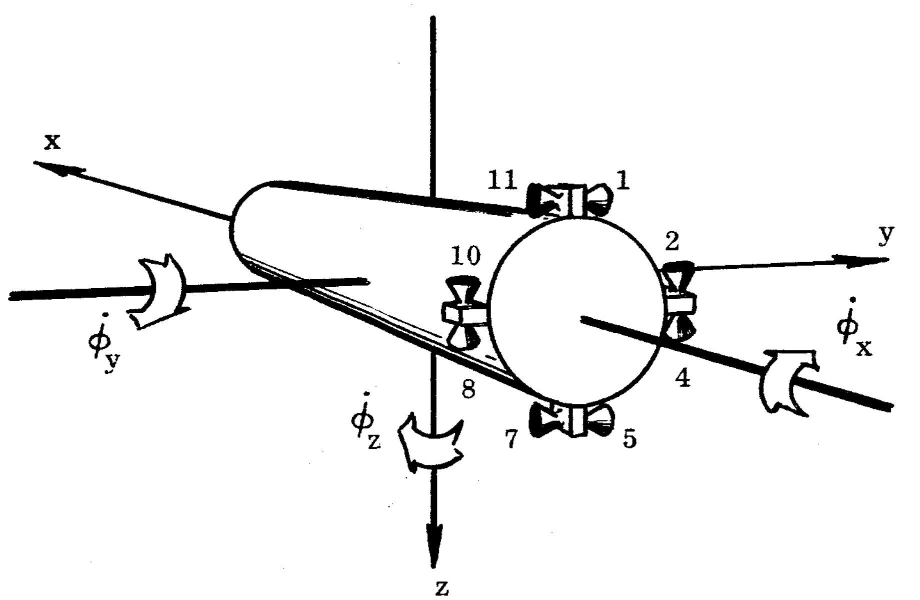
Yaw Axis:

$$\ddot{\phi}_z = \frac{57.3}{I_z} \ell (T_1 + T_5 - T_7 - T_{11}) + \ddot{\phi}_{z_D}$$

Rate product terms have been shown to have negligible influence on the design of the ACS and on the relative inertia of DOTC and conventional systems, hence they have not been included. The terms, $\ddot{\phi}_{x_D}$, $\ddot{\phi}_{y_D}$, $\ddot{\phi}_{z_D}$ represent acceleration disturbances which are of two general types: One type is due to off-nominal vehicle physical charateristics described in the following section, and the other type includes all external "environmental" disturbances due to aerodynamic, gravity gradient and solar influences.



(a) 6 Thruster Configuration



(b) 8 Thruster Configuration

Figure 1. Vehicle and Thruster Configurations

Attitude Engine Thrust Level = 670 N (150 lb)

Minimum Thruster ON Time = 50 ms

Parameter	ISU	English
Pitch/Yaw Inertia	1.5×10^7 kg-m^2	1.1×10^7 slug-ft^2
Roll Inertia	1.5×10^5 kg-m^2	1.1×10^5 slug-ft^2
Pitch/Yaw Moment Arm, ℓ	8 m	26.2 ft
Roll Moment Arm, r	3.6 m	11.8 ft
Pitch/Yaw Angular Acceleration per Engine	0.02 deg/sec^2	0.02 deg/sec^2
Roll Angular Acceleration per Engine	0.91 deg/sec^2	0.91 deg/sec^2
Mass	1.1×10^5 kg	7.5×10^3 slug
Length, ℓ_a	19 m	62 ft
Radius, r_a	3.6 m	11.8 ft

Figure 2. Nominal Parameter Values

2.2.1.2 Off Nominal Vehicle Characteristics

Of significance here are not the deviations experienced in the values of vehicle physical parameters listed in Figure 2, but rather the magnitude of certain physical parameters, important as regards attitude control coupling between axes, which are normally zero. Included are the deviations from nominal in the thruster angular alignment (δ) and in c.g. position (ϵ_y and ϵ_z) from the vehicle center line. The following equations define the approximate incremental acceleration coupling due to these two types of deviations.

$$\Delta\ddot{\phi}_{x_D} = \frac{r\,(57.3)}{I_x}(-T_6\,\delta_6 - T_{12}\,\delta_{12}) + \frac{57.3}{I_x}\Big[+T_6\,\epsilon_y - T_{12}\,\epsilon_y$$

$$+T_1\,\epsilon_z + T_5\,\epsilon_z - T_7\,\epsilon_z - T_{11}\,\epsilon_z\Big]$$

$$\Delta\ddot{\phi}_{y_D} = \frac{\ell(57.3)}{I_y}(-T_1\,\delta_1 - T_5\,\delta_5 + T_7\,\delta_7 + T_{11}\,\delta_{11})$$

$$\Delta\ddot{\phi}_{z_D} = \frac{\ell\,(57.3)}{I_z}(+T_{12}\,\delta_{12} - T_6\,\delta_6)$$

Where: ϵ_x, ϵ_y = c.g. location deviations, positive to the right and up, respectively

δ = angular misalignment in radians of thrusters in a plane normal to the vehicle axis of symmetry, positive for clockwise rotation (aft view)

Similar equations apply for the 8-engine configuration. However, for small misalignments of the thrusters the pitch-yaw thrusters do not couple into the roll axis for the 8 engine configuration. Imperfect matching of thruster pairs will cause some coupling, however.

2.3 ATTITUDE DISTURBANCES DUE TO EXTERNAL ENVIRONMENT

Space vehicle attitude disturbances can arise from any of the following sources:

(1) Aerodynamic

(2) Gravity gradient

(3) Solar pressure

(4) Internal moving parts

(5) Electromagnetic torques

(6) Micrometeorites

(7) Solar flares

A brief evaluation of the last four of these sources indicated that they are normally very small relative to the first three sources, hence were ignored in the subject study. The effects of aerodynamics, gravity gradient and solar pressure are discussed in the following sections.

2.3.1 Disturbances in the Pitch and Yaw Axes

2.3.1.1 Aerodynamic Effects

During the earth orbital portion of space missions, the vehicle can be acted upon by aerodynamic torques with magnitudes which depend upon vehicle inertial and geometric characteristics, orientation of the vehicle to the airstream and atmospheric density, dependent largely upon orbital altitude. Aerodynamic torques, especially on the lower fringe of orbital operations can have a significant influence on the operation of DOTC as well as conventional control systems. Because of the parametric nature of the subject study, however, the gross range of torques is all that is required. These have been estimated for the baseline vehicle assuming it to be a cylinder of the same length (ℓ_a) and radius (r_a) as the baseline vehicle. As for the baseline vehicle, the approximating cylinder is symmetric in pitch and yaw - in fact, it is symmetric about any plane through the longitudinal axis.

As a first step in the development of aero torques, the drag is expressed as follows.

$$D = C_D \, q \, A_p.$$

where:

D = drag force

q = dynamic pressure = $\frac{1}{2}\rho V^2$

ρ = air density

V = vehicle orbital velocity

C_D = drag coefficient = 2.0

A_p = projected vehicle area normal to the velocity vector

$A_p \cong \pi r_a^2 \cos \theta + 2 r_a \ell_a \sin \theta$

θ = angle between the vehicle x body axis and the relative airstream velocity vector

The effective moment arm, d, measured from the aft end of the vehicle, through which the drag force acts is given by

$$d = \left(\frac{\ell_a}{2} - \ell\right) \sin\theta$$

where:

ℓ = distance from aft end of vehicle to the c.g. location

The disturbance acceleration due to drag is then

$$\ddot{\theta} = \frac{C_D\, q\, A_p\, d\, (57.3)}{I} \quad \text{- deg/sec}^2$$

Where I = vehicle moment of inertia in pitch

The steady aerodynamic "bias" acceleration due to a nominally constant vehicle pitch angle, θ, and the "stiffness" term (angular acceleration due to small perturbations, Δ, about the nominal angle) were developed from the following expression:

$$\ddot{\theta}_{aero} = K \sin 2(\theta + \Delta) + \operatorname{sgn}(\theta)\,(K_K)\, \sin^2(\theta + \Delta)$$

where:

$$K = \frac{C_D\, q \left(\frac{\ell_a}{2} - \ell\right) \pi\, r_a^2\, (57.3)}{2I} \quad \text{- deg/sec}^2$$

$$K_K = \frac{C_D\, q \left(\frac{\ell_a}{2} - \ell\right) 2\, r_a\, \ell_a\, (57.3)}{I} \quad \text{- deg/sec}^2$$

For pitch angles from +90 to -90 degrees, the steady bias disturbance acceleration is given by

$$\ddot{\theta}_{bias} = \operatorname{sgn}(\theta) \left[K \left| \sin(2\theta) \right| + K_K \sin^2\theta \right]$$

The small angle "stiffness" disturbance acceleration is:

$$\frac{\ddot{\theta}}{\Delta} = \left[2K \cos(2\theta) + K_K \sin \left| 2\theta \right| \right]$$

Using parameters for the baseline vehicle (see Figures 1 and 2) and in addition:

$$\frac{\ell_a}{2} = \ell = 1.52 \text{ m} = 5 \text{ ft}$$

$$C_D = 2.0$$

$$q_{\substack{184 \text{ km (100 n. mi.)} \\ \text{orbit}}} = 1.68 \times 10^{-2} \text{ N/M}^2 = 3.5 \times 10^{-4} \text{ lb/ft}^2$$

Figures 3 and 4 show the variation in the aerodynamic bias acceleration and aerodynamic "stiffness" for pitch angles from zero to 90 degrees nose down. These aerodynamic disturbance parameters, shown typically for a 184 km (100 n. mi.) orbit in these figures, may be converted to other altitudes by employing the multiplying factors presented in the figures.

2.3.1.2 Gravity Gradient Effects

The angular acceleration in pitch due to gravity gradient torque is approximated for the baseline vehicle by the following equation:

$$\ddot{\phi}_y = (57.3)\ 3/2\ (\dot{\zeta})^2 \sin 2\ \phi_y \ - \text{deg/sec}^2$$

where

$$\dot{\zeta} = \text{vehicle orbital angular rate, rad/sec}$$

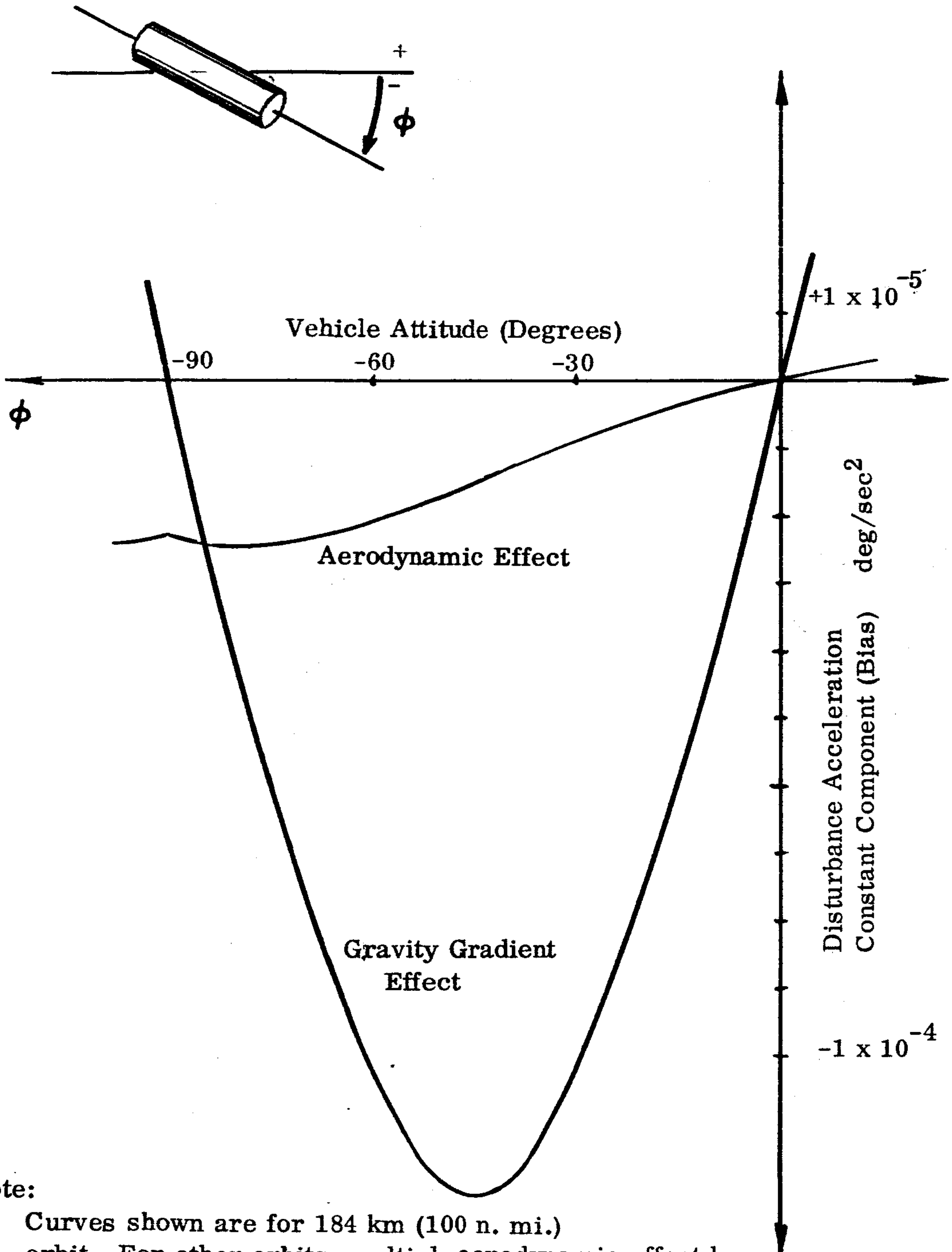
The associated pitch bias moment and small angle "stiffness" term were computed directly from this relation. Setting ϕ_y = nominal pitch angle, ϕ_{y_o}; and $\Delta\phi_y$ = small angle perturbation about this nominal, the bias disturbance acceleration is,

$$\ddot{\phi}_y = \frac{(57.3)\ 3}{2} (\dot{\zeta})^2 \sin 2\ \phi_{y_o} \ - \text{deg/sec}^2,$$

and the "stiffness" term is,

$$\frac{\ddot{\phi}_y}{\Delta\phi_y} = 3(\dot{\zeta})^2 \cos 2\ \phi_{y_o} \ - \frac{\text{deg/sec}^2}{\text{deg}}$$

Figures 3 and 4 show the variation in these parameters with nominal pitch angle for a 184 km (100 n. mi.) orbit. They are not noticeably different for 368 km (200 n. mi.) or 138 km (75 n. mi.).



Note:

Curves shown are for 184 km (100 n. mi.) orbit. For other orbits, multiply aerodynamic effect by:

368 km - 0.03

138 km - 60

Figure 3. Aerodynamic and Gravity Gradient Bias Disturbance Accelerations Versus Pitch Angle.

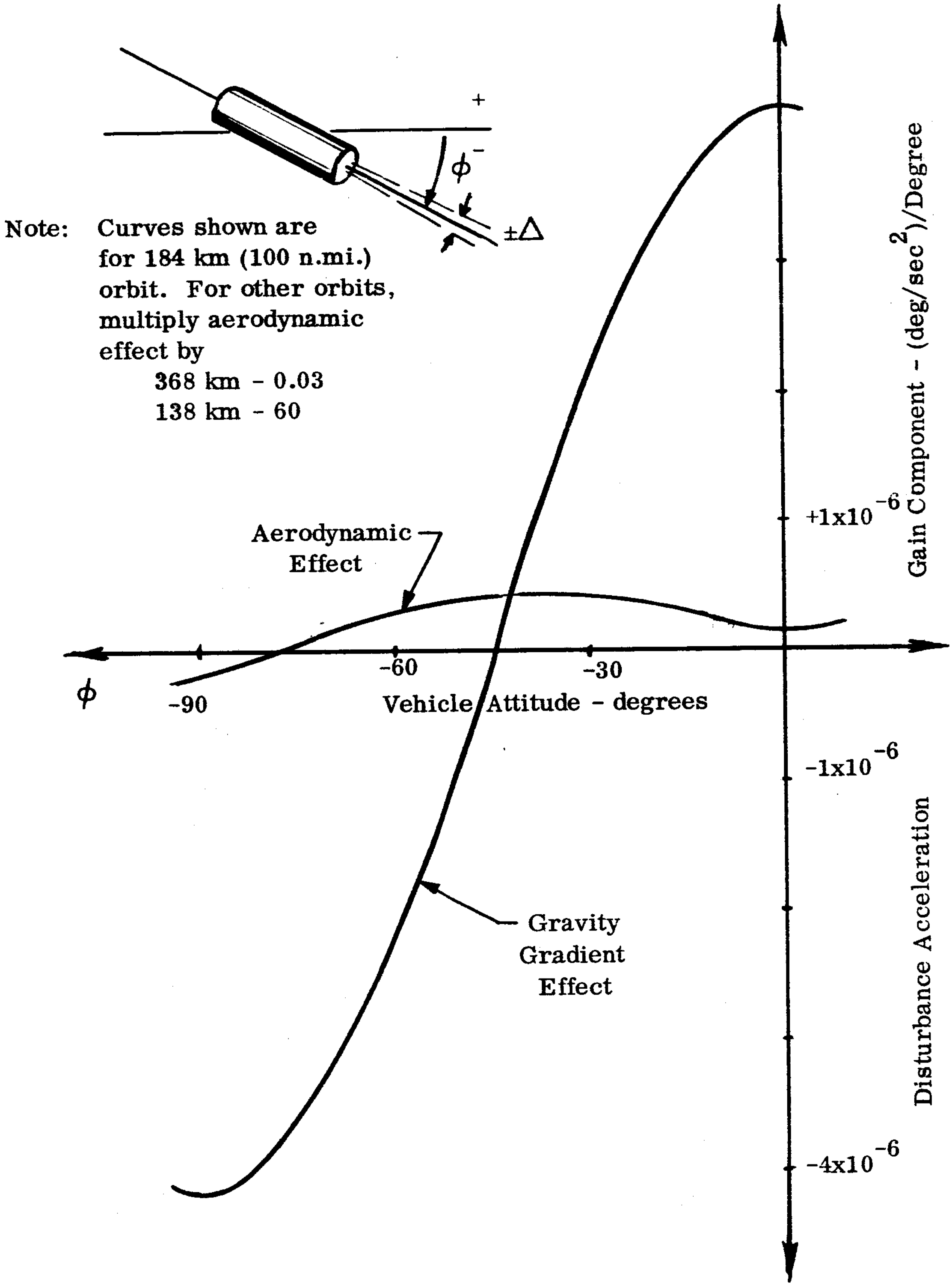


Figure 4. Aerodynamic and Gravity Gradient Stiffness Terms versus Pitch Angle

2.3.1.3 Solar Pressure

In earth orbit, incident radiation from the sun can result in a solar pressure value of about 4.8×10^{-6} N/m^2 (10^{-7} lb/ft^2). This value becomes comparable to aerodynamic "pressure" in the 736 to 920 km (400 to 500 n. mi.) range of altitude. At a 368 km (200 n. mi.) altitude, aerodynamic "pressure" is still more than 100 times the solar pressure value.

Since the basic influence on the vehicle of solar radiation is essentially the same as aerodynamic effects for similar angles of incidence of the relative air stream/ incoming radiation, consideration for the full magnitude range of aerodynamic moments encompasses the solar radiation effects. Hence, solar pressure effects are not considered separately.

2.3.2 Disturbances in the Roll Axis

For vehicles of the general baseline configuration class (Figure 1), there are no significant rolling moment disturbances generated directly by external influences such as aerodynamics, solar pressure and gravity gradients.

2.4 DISTURBANCES DUE TO CONTROL COUPLINGS

2.4.1 Control Coupling Disturbances in Pitch and Yaw

For either of the baseline thruster configurations (Figure 1a and 1b) control moments applied in the pitch axis can, due to thruster misalignments and/or c.g. position deviations, cause yaw axis moments and vice versa. This type of coupling is largely inconsequential since, for a one degree thruster misalignment, the reflected moment in the unwanted axis is only 1/57.3 of the applied moment. However, as discussed in the following section, the coupling from pitch or yaw control moments into the roll axis can become very significant for situations where bias type moments are existent in the pitch and/or yaw axes.

2.4.2 Disturbances in Roll

Rolling moment disturbances can be generated by the pitch and/or yaw thruster firings if these units are misaligned so that their line of thrust action does not pass through the vehicle centerline, on which the vehicle c.g. is assumed located. Alternatively, if the thrust passes through this centerline and the c.g. is offset from it, a roll moment is generated by the thrust application.

For purposes of the subject DOTC program, and in particular for estimating the control coupling from pitch and/or yaw into roll, it is important to estimate the activity of the pitch/yaw thrusters which will range from undisturbed limit cycle operation to "one-sided" thruster firing to counter large bias type moments caused, for example, by aerodynamic influences.

Studies have shown that the influence of pitch/yaw control on the roll axis is insignificant for reasonable thruster misalignments (one degree or less) unless large pitch/yaw "bias" disturbances are present. In estimating the activity of the pitch/yaw thrusters it has been assumed that the pitch axis experiences the large disturbance at any given time. This is conservative from the standpoint that a much greater roll disturbance is imparted by misalignment of the pitch thrusters than from the yaw thrusters (see Figure 1a).

Using pitch disturbances ranging from 10^{-4} deg/sec^2 to 10^{-5} deg/sec^2, Figure 5 presents the resulting limit cycle activity of the pitch system as well as the associated disturbance imparted to the roll axis. As apparent from Figure 3, this range of pitch disturbances includes the largest that can be experienced for the reference vehicle in a 184 km (100 n. mi.) or greater altitude orbit (10^{-4} deg/sec^2).

The lower two graphs show the period and peak-to-peak amplitude of the pitch oscillations versus thruster on time, for three different values of r, the ratio of control acceleration to disturbance acceleration. The upper graph shows the angular rate bit induced into roll, as a function of pitch thruster on time.

It is noteworthy that for pitch on times less than about 0.7 second, the induced roll bit is less than the typical DOTC roll bit, equivalent to about 10 milliseconds. In this range, the roll disturbance consists of relatively small pulses of a relatively high frequency, and hence may be approximated as a continuous disturbance moment. For pitch on times greater than 0.7 second, the disturbance tends to consist of large, relatively infrequent, pulses and thus may be treated as a transient or initial condition. The logic concept evaluation studies of Section 5.0 treat both constant disturbances and initial conditions in general, and hence cover the majority of disturbances coupled from pitch (and yaw):

(1) When pitch disturbance moments are very large (order of 10^{-4} deg/sec^2) the pitch limit cycle period is 200 seconds or less and the impulse disturbance to roll is about $(1.4)10^{-3}$ deg/sec or less. Though this period is much less than roll, the impulse imparted to roll is on the same order as the roll limit cycle rate.

(2) At intermediate pitch disturbance levels the pitch limit cycle period is still several times lower than roll and the associated impulse disturbances into roll are small compared to the roll limit cycle rate.

(3) At low pitch disturbance levels the pitch limit cycle period is on the order of or greater than roll. The disturbance levels into roll are then very small and probably inconsequential compared with the normal roll limit cycle rate.

2.5 THRUSTER CHARACTERISTICS

The DOTC study included approximations to thruster turn-on and turn-off dynamic response characteristics. A typical thrust transient time history following application of voltage to the propellant valve coil consists of a time delay, due to valve dynamics and injector fill time, followed by a rapid time rise of chamber pressure to the steady state value. When the command voltage is removed from the propellant valve coil, the thrust level remains at the steady state value for a brief delay time, then decays to zero thrust level. This typical thrust response characteristic is illustrated in Figure 6. These thruster dynamic effects were approximated in the subject study by an equivalent "turn-on" time delay followed by a step rise in thrust to full value. Similarly, valve closing dynamics and chamber pressure decay dynamics were approximated by an equivalent "turn-off" time delay, τ_D. Figure 6 illustrates the approximations employed for the thrust chamber dynamics.

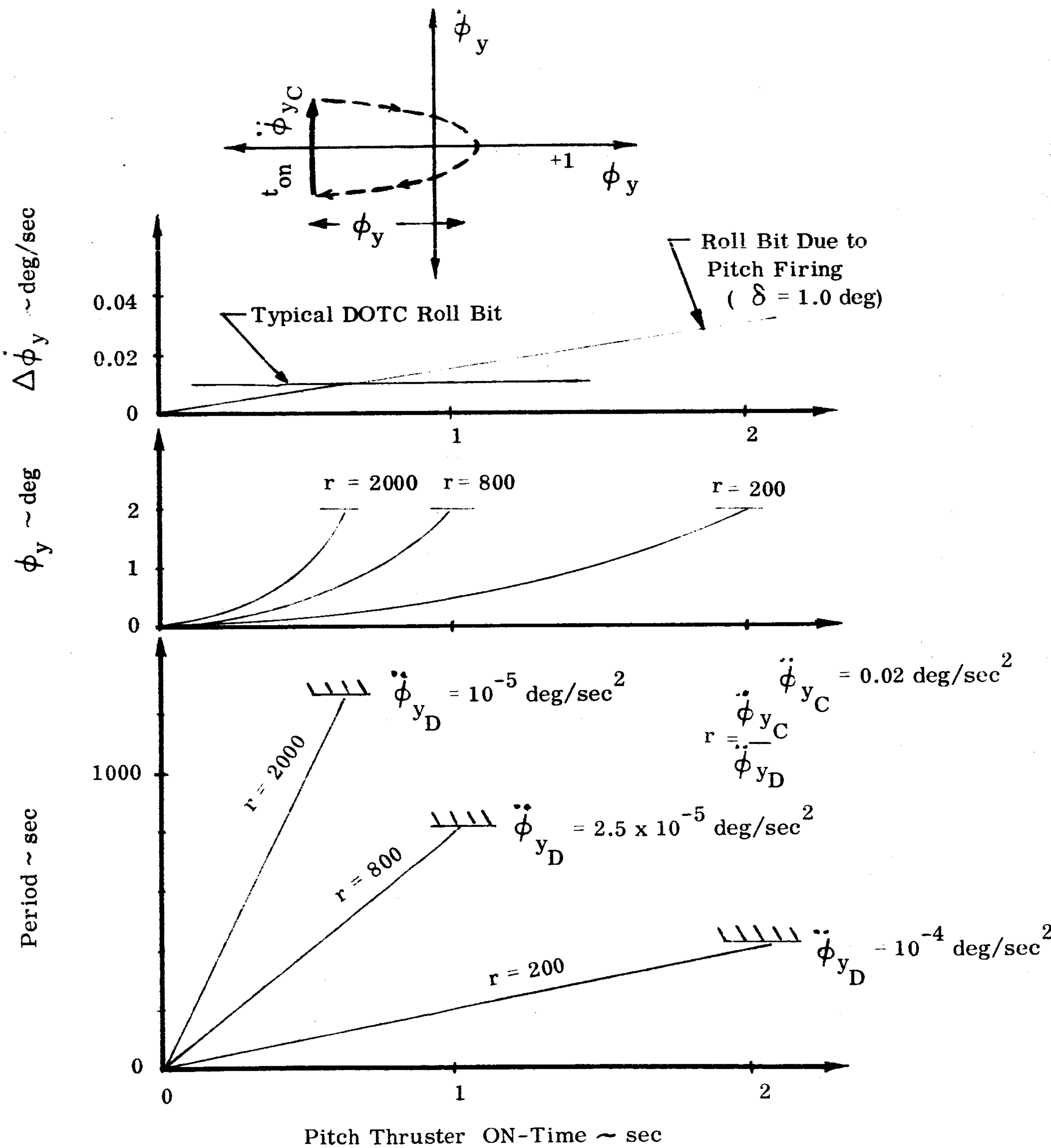
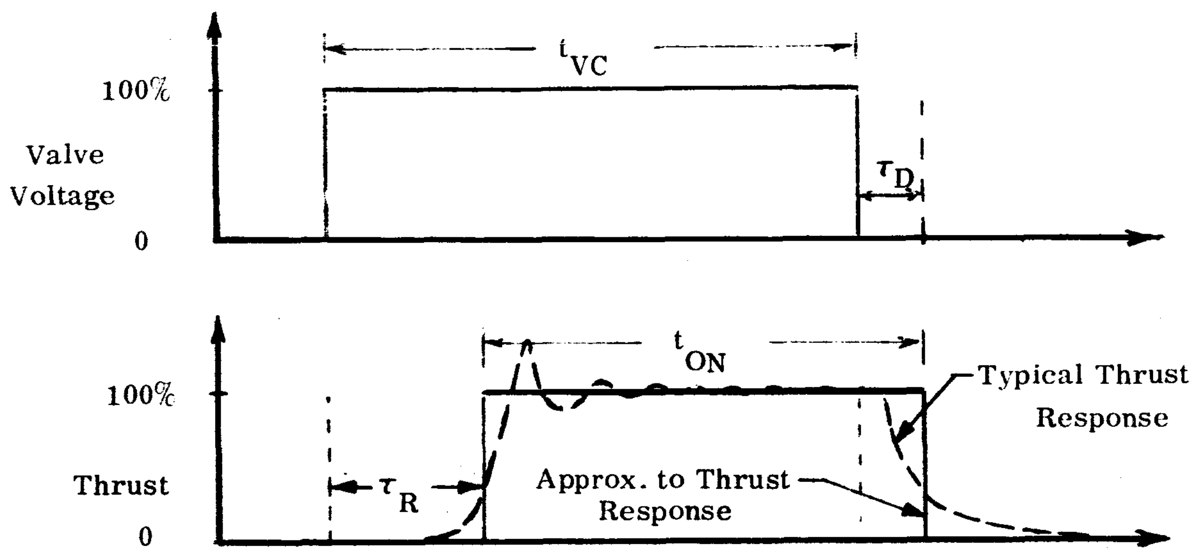


Figure 5. Pitch Axis Limit Cycle Characteristics in Presense of Bias Disturbances



t_{VC}	-	Time duration of valve voltage command signal
τ_R	-	Delay time equivalent to valve open delay and thrust rise characteristic
τ_D	-	Delay time equivalent to thrust decay characteristic and valve close delay time
t_{ON}	-	Equivalent square wave thruster on-time
t_{ON}	-	$t_{VC} + \tau_D - \tau_R$

Figure 6. Approximation to Thruster Transient Response

3.0 DEVELOPMENT OF CONTROL LOGIC CONCEPTS

The various Differential On-Time Control logic concepts which were formulated and studied during this program are described in this section. The principal alternatives involved in the synthesis of DOTC concepts are described first. Conventions and definitions are then presented which are used in subsequent detailed descriptions of four major DOTC logic concepts.

A description of the Linear Signal Mixing control concept, which was used as the primary basis of comparison for system performance, is also presented in this section.

3.1 DOTC LOGIC CONCEPTS

3.1.1 General

DOTC concepts can be categorized on the basis of two major attributes: the thruster pairs used for differential firings, and the method used to determine and execute the required firing times of each thruster. Where the chosen pair is fired conventionally for control of the axis of large inertia and differentially for control of the axis of low inertia (as in the case of Differential Yaw Thrusting, decribed below), DOTC concepts may be further categorized on the basis of which axis is given priority when errors in both axes occur simultaneously.

3.1.1.1 Thruster Pair Selection

For the baseline vehicle configuration, the small impulse bits provided by DOTC methods are required in only the roll (low inertia) axis. This differential control action may be achieved either by (1) firing for unequal times two back-to-back thrusters which cancel one another in roll moment effect when both are firing (Differential Back-to-Back, abbreviated DBB) or (2) by firing two thrusters selected so that their roll components tend to cancel, while their yaw components add (Differential Yaw Thrusting, abbreviated DYT).

In the latter case, the same pair of thrusters used for DOTC in roll is fired when yaw corrections are required. This, in fact, provides the principal advantage of DYT - a roll correction can be achieved virtually free whenever a yaw firing is made, by simply firing one thruster longer than the other. However, a problem is created whenever both yaw and roll errors exist simultaneously. The simplest form of DOTC logic gives priority to the yaw axis because it requires that the yaw error be completely corrected before turning off one thruster and thus initiating roll control by the differential action. Because of the low control acceleration in yaw, large roll errors may build up following a transient disturbance which imparts large rates to both axes. The logic can be modified, however, to give priority to roll corrections by firing only one thruster until roll is corrected, and then firing a second one to control yaw. This roll priority DOTC concept is advantageous. Not only is roll corrected quickly by virtue of the large roll control acceleration available, but yaw is corrected with at least half power at all times.

DOTC logic concepts were developed for DBB, DYT/Roll Priority, DYT/Yaw Priority, and a combination of DBB and DYT/Yaw Priority. They are described in more detail later in this section.

3.1.1.2 Differential Firing Time Methods

A pair of thrusters may be differentially fired in either an open loop or a closed loop basis. In the former case, the firing time of each thruster, and hence the differential firing time, is determined before the firings are initiated and is executed in a programmed manner. This may be accomplished by turning both thrusters on simultaneously and turning them off differentially, turning them on differentially and turning them off simultaneously, or turning them both on and off differentially. Whichever sequence is used, the error or uncertainty of the resultant differential firing is equal to the difference in errors of the individual firings. Hence, the inaccuracy of the achieved differential firing time reflects the uncertainties of the firing time commands, the thruster turn-on dynamics, the steady state thrust levels, and the thrust turn-off dynamics.

Most of these errors can be circumvented by closed loop control of the thruster firing times. In this approach, the turnoff of the second thruster is commanded when the roll error is sufficiently reduced. The uncertainties of all previous events are thus accounted for and only the uncertainty in the turn-off dynamics of the last thruster will produce errors. An additional advantage of this method of achieving differential firing times is that it can be conveniently implemented with threshold logic. When a roll error threshold is exceeded, two thrusters are commanded on. In the case of DBB, one thruster is turned off after it has fired for its minimum on-time, and the other is left on until the roll error is brought within its threshold. In the case of DYT, the first thruster may be turned off either on the basis of time, as above, or allowed to fire until any yaw error that may exist is corrected.

Because the closed loop approach offers both performance and logic simplicity advantages, it was selected for use in all DOTC concepts investigated under this study.

3.1.2 Conventions and Definitions

The following DOTC logic development is presented in terms of the notations and techniques of Boolean algebra.

The vehicle configuration, thruster designation, attitude angle sign convention and coordinate system are shown in Figure 1.

Figure 7 presents the overall control loop and identifies the error signal and angular rate thresholds which are used in the DOTC system formulation. The logical variables (threshold outputs) which were employed in the study are also defined by Figure 7. These variables are either 1 or 0 (i.e., true or false). The control system logical variables are further defined in terms of the vehicle roll and yaw phase plane diagrams of Figure 8. The ordinates of Figure 8 represent the vehicle angular rates and the abscissae represent the vehicle attitude angles. The vehicle angular rate and position trajectories resulting from the firing of various thrusters are indicated in these diagrams. Further definition and clarification of the regions of the roll and yaw phase planes for which various system logical variables are "true" or "high" are presented in Figures 9 and 10.

3.1.3 Development of Equations for a DOTC - DYT - YP Concept

This DOTC concept (without the P and O thresholds) was the first approach studied, and is one of the simplest. The P and O thresholds were incorporated after an analog computer simulation revealed the possibility of a hang-up, or rapid limit cycling, near the roll thresholds of the original systems.

Four possibilities with regard to the roll and yaw error signals are the following:

(a) Yaw error exceeds its threshold value and roll error does not

(b) Roll error exceeds its threshold value and the yaw error does not

(c) Both the roll and yaw error signals exceed their thresholds

(d) Neither error exceeds its threshold

For this DOTC - DYT - YP concept, the control action in response to the possible threshold situations is as follows:

(I) If the yaw threshold signals C or D go high, command firing of the appropriate pair of thrusters. When yaw error has been driven within its threshold ($C = O$, $D = O$), and the yaw rate has changed sign, command thrust termination in one of two ways.

- (a) If roll signals P and O are not high ($P = O$ and $O = O$), command simultaneous shutoff of both thrusters.
- (b) If either the P or O signal is high, shut off the thrusters differentially so as to reverse the sign of the roll rate. The thrust shutoff command of one thruster will be subject to the above stated yaw axis conditions ($C = O$ or $D = O$ and $C_R = 1 \rightarrow C_R = O$ or $D_R = 1 \rightarrow D_R = O$). The thrust termination command for the other thruster results when the

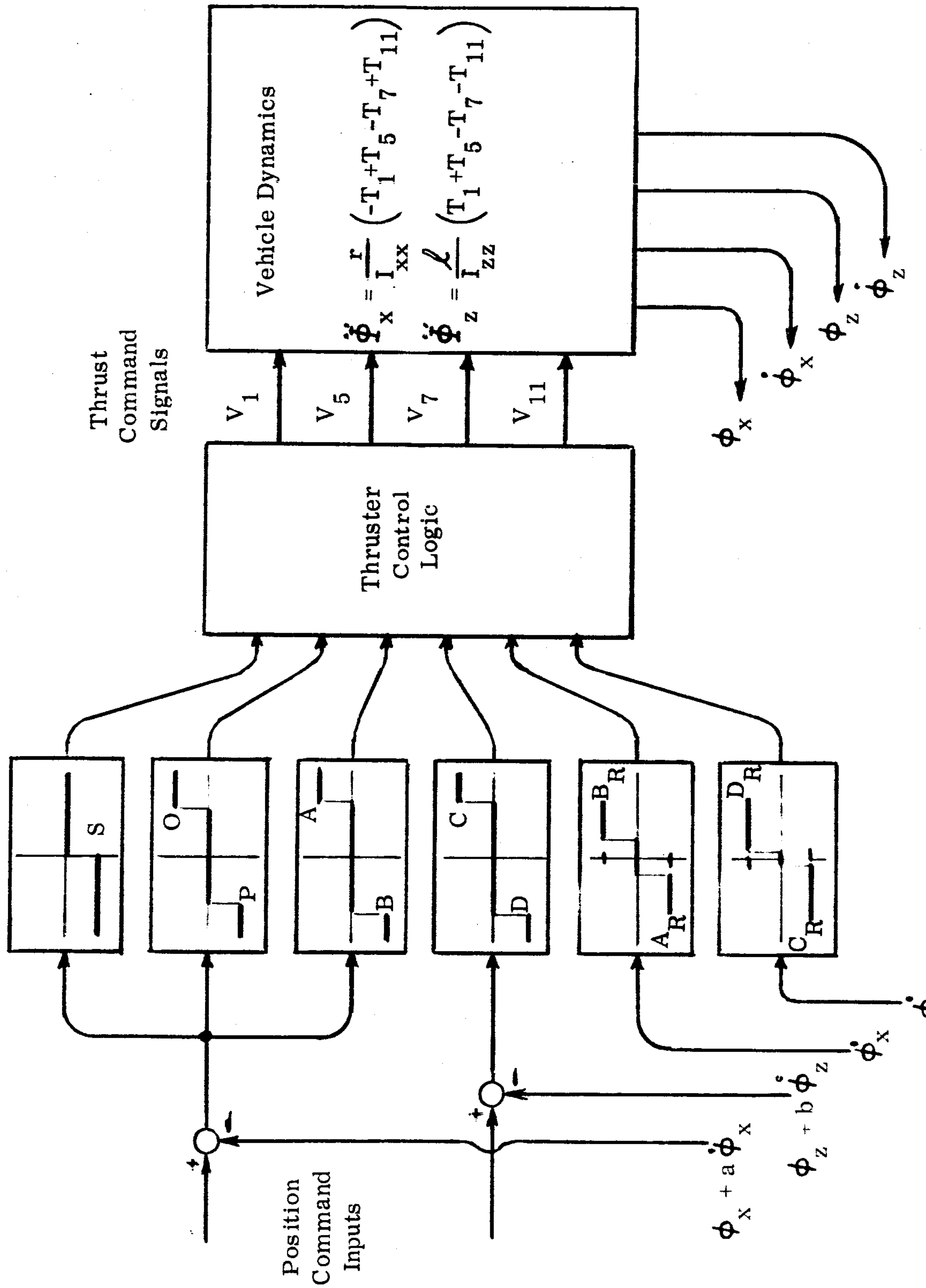


Figure 7. Definition of Thresholds Used to Obtain DOTC Variables

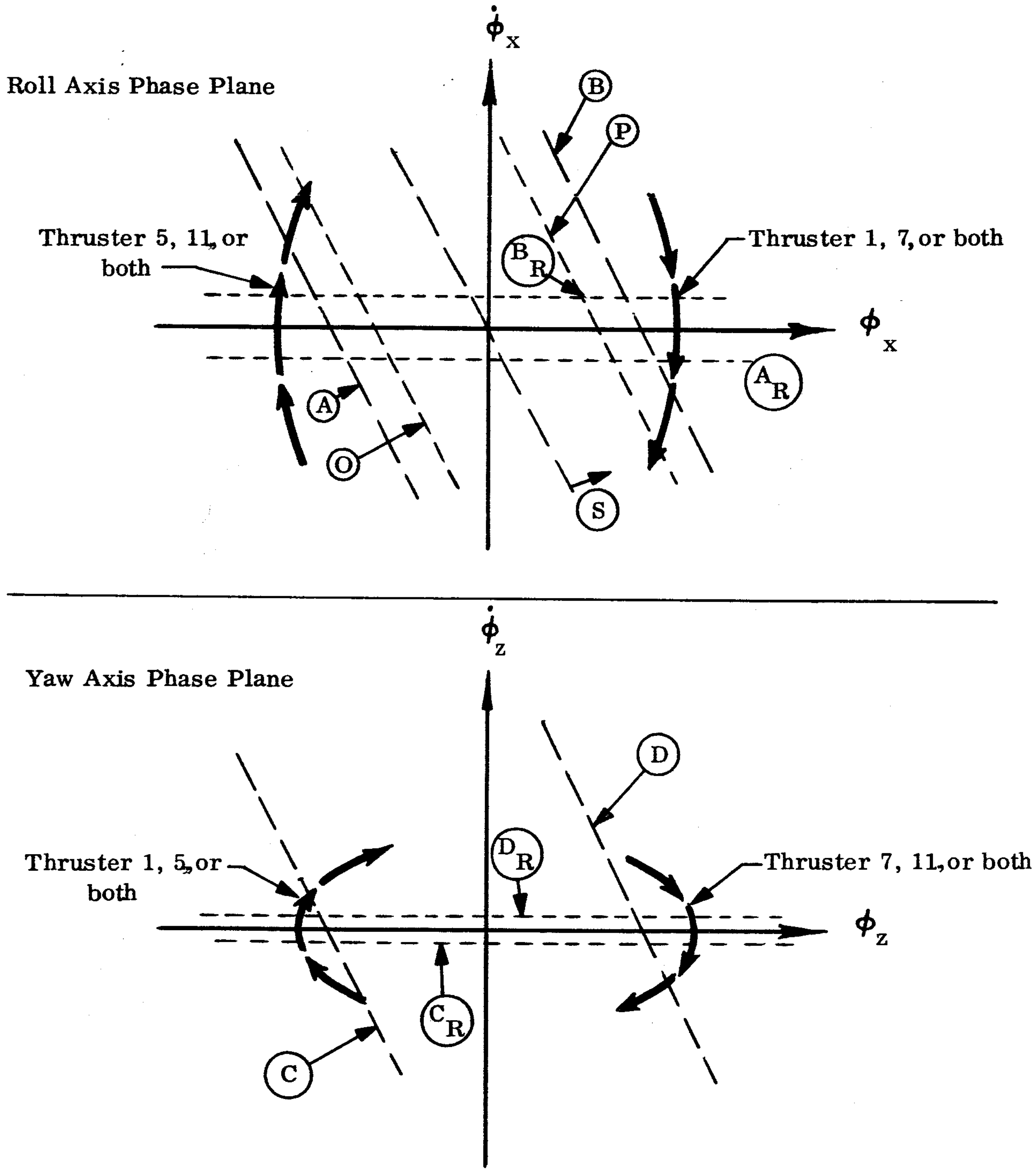
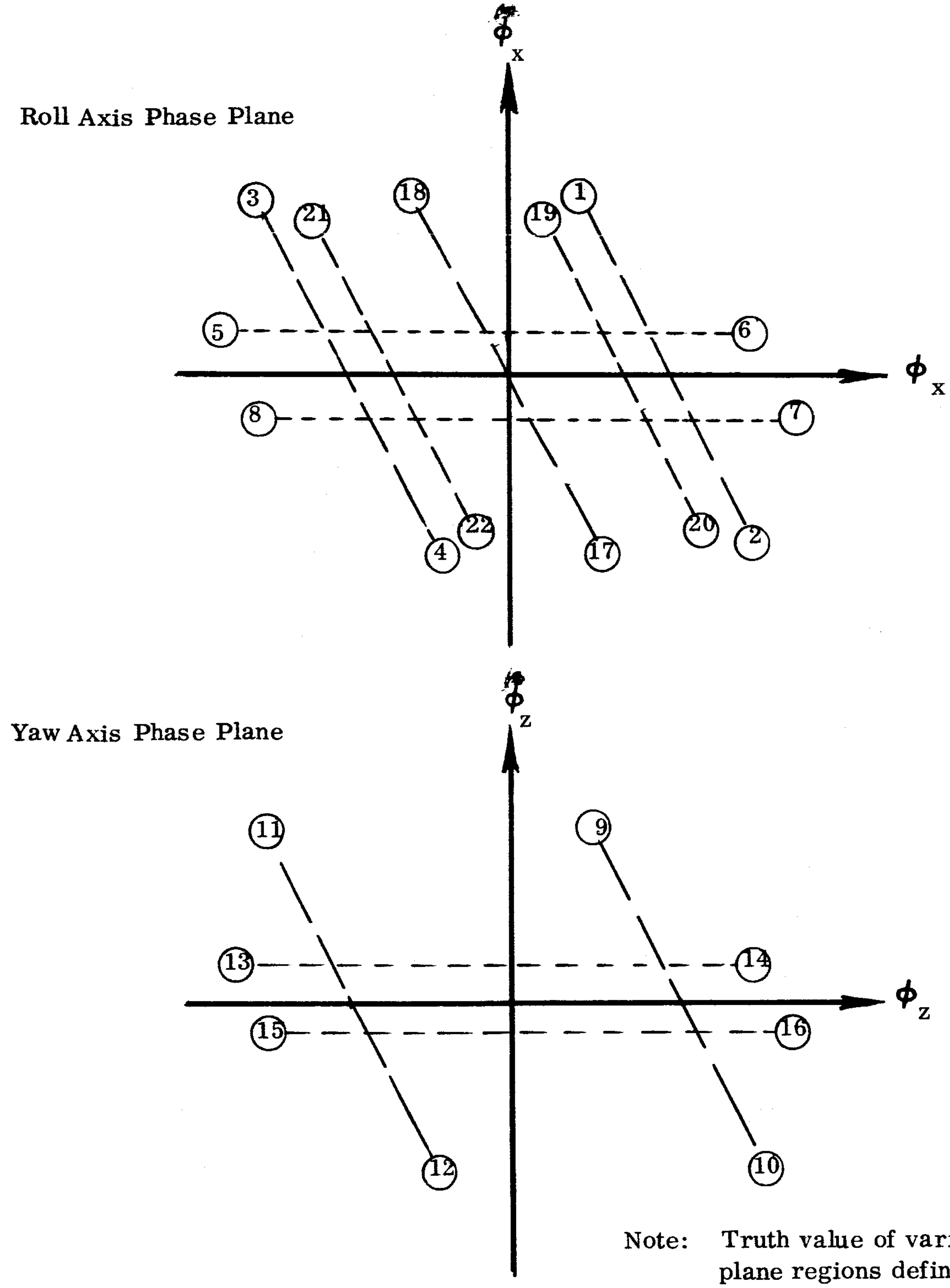


Figure 8. Location of DOTC Variable Switching Lines in Roll and Yaw Phase Planes



Note: Truth value of various phase plane regions defined in Figure 10.

Figure. 9. DOTC Variable Switching Line Designations in Roll and Yaw Phase Planes

A is "high" to left of line	3 — 4
B is "high" to right of line	1 — 2
C is "high" to left of line	11 — 12
D is "high" to right of line	9 — 10
A_R is "high" in region below line	7 — 8
B_R is "high" in region above line	5 — 6
C_R is "high" in region below line	15 — 16
D_R is "high" in region above line	13 — 14
S is "high" to right of line	17 — 18
P is "high" to right of line	19 — 20
O is "high" to left of line	21 — 22

Note: Figure 9 shows location of switching lines referred to above.

Figure 10. Definition of Truth Value of DOTC Variables in Various Regions of Phase Planes

roll axis error is within its threshold and the roll rate threshold signals are down ($A_R = 1 \rightarrow A_R = O$ or $B_R = 1 \rightarrow B_R = O$).

(II) If the roll threshold is exceeded, A or B go high, and no yaw correction is needed, fire two thrusters which will reverse the sign of the yaw rate ($C_R = 1 \rightarrow C_R = O$ or $D_R = 1 \rightarrow D_R = O$). After the yaw rate has changed sign, terminate thrust differentially as described for situation (I) (b).

(III) If both the yaw and roll thresholds are exceeded, follow the rules of situation (I) for both turn-on and turn-off.

The control action rules stated above are rigorously expressed in the form of thruster command equations using Boolean notation in Figure 11. To aid in understanding this logic, the build-up of a typical equation is described below.

The "ON" conditions for thruster number 1, from (I) and (II), is

$$(V_1)_{ON} = C + C_R \cdot (A + B)$$

Incorporating the first "OFF" condition of (I)

$$V_1 = C + C_R \cdot (A + B + V_1)$$

Incorporating the second "OFF" condition of (I)

$$V_1 = \left[C + C_R \cdot (A + B + V_1) + (B + B_R \cdot P) \cdot V_1 \right]$$

In order to protect against possible back-to-back firing a final term is added.

$$V_1 = \left[\bar{V}_7 \cdot \bar{V}_{11} \right] \cdot \left[C + C_R \cdot (A + B + V_1) + (B + B_R \cdot P) \cdot V_1 \right]$$

3.1.4 Development of the DOTC - DYT Roll Priority Logic Equations

A system employing the control logic developed in the foregoing and presented in Figure 11 could exhibit rather large transient roll errors if large yaw and roll maneuvers are called for simultaneously. A modified control logic that permits any roll axis control requirement to take priority is formulated below.

The control action in response to the possible threshold situations is stated as follows:

(I) If the roll thresholds are not exceeded and a yaw error threshold is exceeded, turn on two appropriate thrusters. Thrust termination occurs in one of the two following ways.

$$V_1 = (\bar{V}_7 \cdot \bar{V}_{11}) \cdot \left[C + (C_R \cdot (A + B + V_1)) + ((B + B_R \cdot P) \cdot V_1) \right]$$

$$V_5 = (\bar{V}_7 \cdot \bar{V}_{11}) \cdot \left[C + (C_R \cdot (A + B + V_5)) + ((A + A_R \cdot O) \cdot V_5) \right]$$

$$V_7 = (\bar{V}_1 \cdot \bar{V}_5) \cdot \left[D + (D_R \cdot (A + B + V_7)) + ((B + B_R \cdot P) \cdot V_7) \right]$$

$$V_{11} = (\bar{V}_1 \cdot \bar{V}_5) \cdot \left[D + (D_R \cdot (A + B + V_{11})) + ((A + A_R \cdot O) \cdot V_{11}) \right]$$

Figure 11. DOTC-DYT Yaw Priority Control Logic Equations

(a) If roll thresholds P and O are not exceeded ($P = O, O = O$), terminate both thruster firings when the yaw error has been driven within the threshold ($D = O = C$).

(b) If a roll threshold is exceeded, turn off the thrusters differentially. One thruster is turned off in accordance with (a) above, and the other is turned off when $P = O$ or $O = O$ and $A_R = 1 \rightarrow A_R = O$ or $B_R = 1 \rightarrow B_R = O$.

(II) If a roll threshold is exceeded apply turn on pulses to two thrusters. Select the thrusters such that the existing yaw rate will be reduced. One of these selected thrusters will burn for the minimum on-time, the other will remain or until the roll error is corrected. ($A = O = B, A_R = 1 \rightarrow A_R = O, B_R = 1 \rightarrow B_R = O, P = O = O$).

The foregoing rules are expressed in form of Boolean equations.

From the "ON" conditions described in (I) and (II).

$$(V_1)_{ON} = C \bullet (\bar{A} \bullet \bar{B}) + \left[C_R \bullet (A + B)\right]^*$$

The "asterisk" signal is a short duration pulse signal — for example, the output of a "Single Shot".

Incorporating the "OFF" conditions of (I) and (II)

$$V_1 = \left[(C \bullet \bar{A} \bullet \bar{B}) + (C_R \bullet (A + B))^* + (B + B_R \bullet P) \bullet V_1\right]$$

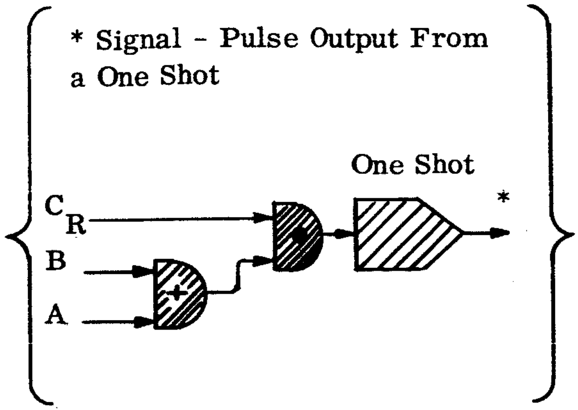
Also, a term to prevent back to back thruster firings is provided.

$$V_1 = (\bar{V}_7 \bullet \bar{V}_{11}) \bullet \left[(C \bullet \bar{A} \bullet \bar{B}) + (C_R \bullet (A + B))^* + (B + B_R \bullet P) \bullet V_1\right]$$

Figure 12 presents the complete set of control logic equations.

3.1.5 Development of DBB - DOTC Logic

The previously developed DOTC concepts are based on controlling the roll axis by differential turn off of a pair of yaw thrusters. In order to satisfy roll axis requirements, these DOTC concepts can produce premature yaw firings; i.e., firings not required, based on the yaw error threshold limits. Therefore, the following DOTC logic was evolved, based on control of the roll axis by differential back-to-back (DBB) thruster firings.



$$V_1 = (\bar{V}_7 \cdot \bar{V}_{11}) \cdot \left[(C \cdot \bar{A} \cdot \bar{B}) + (C_R \cdot (A + B))^* + (B + B_R \cdot P) \cdot V_1 \right]$$

$$V_5 = (\bar{V}_7 \cdot \bar{V}_{11}) \cdot \left[(C \cdot \bar{A} \cdot \bar{B}) + (C_R \cdot (A + B))^* + (A + A_R \cdot O) \cdot V_5 \right]$$

$$V_7 = (\bar{V}_1 \cdot \bar{V}_5) \cdot \left[(D \cdot \bar{A} \cdot \bar{B}) + (D_R \cdot (A + B))^* + (B + B_R \cdot P) \cdot V_7 \right]$$

$$V_{11} = (\bar{V}_1 \cdot \bar{V}_5) \cdot \left[(D \cdot \bar{A} \cdot \bar{B}) + (D_R \cdot (A + B))^* + (A + A_R \cdot O) \cdot V_{11} \right]$$

Figure 12. DOTC-DYT Roll Priority Control Logic Equations

The control action in response to possible threshold situations for this DBB-DOTC concept is described below.

(I) If the yaw threshold is exceeded ($C = 1$ or $D = 1$) and the roll threshold is not exceeded ($A = O$, $B = O$), turn on two appropriate thrusters in order to correct yaw.

(II) If a yaw threshold and a roll threshold are exceeded, turn on one appropriate thruster to correct roll and yaw.

(III) If the roll threshold is exceeded and the yaw threshold is not exceeded, correct roll by back-to-back thrusting. The thruster selection and turn off rules are as follows.

(a) The thruster pair is selected such that the differential shutoff results in the required roll correction and also produces a reduction in the yaw rate. The selected thrusters are pulsed on when the roll threshold is exceeded. One of the selected thrusters will burn for its minimum on-time.

(b) The other thruster will fire and apply roll correction until the roll threshold is no longer exceeded and the roll rate threshold limits are passed.

The above described strategy was expressed in Boolean equation form. The equation for thruster number one is formulated below.

From (I) and (II),

$$V_1 = (B \bullet C) + (C \bullet \bar{A} \bullet \bar{B})$$

The terms corresponding to (III)(a) and (III)(b) are,

$$V_1 = (\bar{C} \bullet \bar{D}) \bullet \left[(B \circ C_R + A \circ D_R)^* + (B + B_R \bullet \bar{A}) \circ V_1 \right]$$

Combining the above terms for thruster number one,

$$V_1 = (B \bullet C) + (C \circ \bar{A} \circ \bar{B}) + (\bar{C}\,\bar{D}) \bullet \left[(B \circ C_R + A \bullet D_R)^* + (B + B_R \bullet \bar{A}) \circ V_1 \right]$$

Figure 13 shows the complete set of control logic equations for the DBB - DOTC concept described above.

$$V_1 = (B \cdot C) + (C \cdot \bar{A} \cdot \bar{B}) + \left[(B \cdot C_R + A \cdot D_R)^* + (B + B_R \cdot \bar{A}) \cdot V_1 \right] \cdot (\bar{C} \cdot \bar{D})$$

$$V_5 = (A \cdot C) + (C \cdot \bar{A} \cdot \bar{B}) + \left[(B \cdot D_R + A \cdot C_R)^* + (A + A_R \cdot \bar{B}) \cdot V_5 \right] \cdot (\bar{C} \cdot \bar{D})$$

$$V_7 = (B \cdot D) + (D \cdot \bar{A} \cdot \bar{B}) + \left[(B \cdot D_R + A \cdot C_R)^* + (B + B_R \cdot \bar{A}) \cdot V_7 \right] \cdot (\bar{C} \cdot \bar{D})$$

$$V_{11} = (A \cdot D) + (D \cdot \bar{A} \cdot \bar{B}) + \left[(B \cdot C_R + A \cdot D_R)^* + (A + A_R \cdot \bar{B}) \cdot V_{11} \right] \cdot (\bar{C} \cdot \bar{D})$$

*Signal – pulsed output from a ONE SHOT

Figure 13. DOTC-DBB Control Logic Equations

3.1.6 Development of a Combination DBB-DYT-DOTC Concept

The experience gained with the DYT-DOTC concepts and the previously discussed DBB-DOTC concept served as the starting point from which a final combined DBB-DYT-DOTC concept was evolved. The DBB-DOTC logic was simulated with a digital computer and tested under various conditions. As additional desirable features of the original DBB logic became evident, it was modified to improve its performance.

The control action of the final DOTC concept in response to various threshold error states is presented below:

(I) If a yaw threshold is exceeded, ($C = O \rightarrow C = 1$ or $D = O \rightarrow D = 1$), and if the roll error thresholds are not exceeded, $A = O$ and $B = O$, then uncoupled yaw thrusting with two engines firing simultaneously will be commanded. This uncoupled yaw control action will be terminated in one of the following ways:

  (a) If the yaw error is driven within its threshold ($C = O$ and $D = O$) and the yaw rate changes sign ($C_R = 1 \rightarrow C_R = O$ or $D_R = 1 \rightarrow D_R = O$) and the roll thresholds are still not exceeded and $(B_R \bullet \bar{S}) = O$ and $(A_R \bullet S) = O$, then both engines are commanded to stop firing simultaneously. If $(B_R \bullet \bar{S}) = 1$ or $(A_R \bullet S) = 1$, then differential yaw shutoff will occur such that $B_R = 1 \rightarrow B_R = O$ or $A_R = 1 \rightarrow A_R = O$.

  (b) If during the uncoupled corrective yaw thrusting, the roll error exceeds its threshold value ($A = 1$ or $B = 1$), then one of the two firing thrusters is shut off. Thruster selection is such that the remaining single thruster produces corrective action in both yaw and roll.

(II) If both the yaw error and the roll error are outside their respective deadbands ($C = 1$ or $D = 1$ and $A = 1$ or $B = 1$), then coupled yaw-roll control thrusting with a single thruster firing will be commanded. If the roll error is reduced to within its threshold ($A = O$ and $B = O$), and the roll rate that existed at thrust startup is reduced past its threshold value $A_R = 1 \rightarrow A_R = O$ and $B_R = O \rightarrow B_R = 1$ or $B_R = 1 \rightarrow B_R = O$ and $A_R = O \rightarrow A_R = 1$ or the roll error changes sign $S = 1 \rightarrow S = O$ or $S = O \rightarrow S = 1$, then the roll/yaw corrective action is modified in one of two ways:

  (a) If the yaw error has also been reduced to within its threshold ($C = O$ and $D = O$) while the above described roll control action was taking place, the single firing thruster is shut off.

  (b) If the yaw error is outside its control deadband ($C = 1$ or $D = 1$) at the time the above described roll control action is terminated, a second yaw control thruster is commanded to fire such that uncoupled yaw control is obtained. When yaw error is within the threshold, both thrusters are commanded off.

(III) If the yaw error threshold is not exceeded ($C = O$ and $D = O$) and if the roll error threshold is exceeded ($A = 1$ or $B = 1$), then back-to-back thrusting will be commanded.

A pair of thrusters is selected such that differential shutoff results in the desired roll correction and also contributes to reducing an existing yaw rate. Both thrusters are commanded to start firing at the same time. One engine is commanded on for the minimum firing time. The other thruster remains on until the roll error signal is driven within its threshold and $B_R = 1 \rightarrow B_R = O$ or $A_R = 1 \rightarrow A_R = O$.

(IV) A mode control signal, Q, is included in the system. This mode control signal is used to provide for the possibility of large disturbances and/or commands. Setting $Q = 1$ will render the back-to-back feature of the control logic inoperative, and will enable alternate single engine ON conditions for obtaining roll corrective action. These alternate single engine ON conditions are the following:

If $Q = 1$, and if the roll error threshold is exceeded ($A = 1$ or $B = 1$), single engine firing will be commanded, and the engine selection will be such that the corrective roll action tends to maintain small yaw axis drift rates. This is accomplished in the following way.

The selection of the initial engine to fire is made on the basis of correcting the roll error and driving the existing yaw rate toward zero. When the first yaw rate threshold is passed, a second engine begins to correct the roll error and no more acceleration is produced in the yaw axis and therefore, small yaw rates are maintained. Engine shutoff will be effected when the roll error is within its threshold ($A = O$ and $B = O$). For the condition where the yaw rate is very small ($C_R = O$ and $D_R = O$), the Q-monitored logic will permit two engines to fire to provide pure roll action.

The foregoing thrust command rules can be expressed in Boolean equation form.

The formulation of the control logic equation for thruster number one is given below. Based on Figures 3 to 10 and conditions (I) through (IV) above, the following step-by-step formulation is presented.

(1) ON-condition (I) and OFF-condition (I, a)

$$V_{1_{(1)}} = (C + C_R \circ V_1) \circ \bar{B} \bullet \bar{A} + B_R \circ S \bullet V_1$$

(2) OFF-condition (I)(b) is incorporated as follows:

$$V_{1_{(2)}} = (C + C_R \bullet V_1) \bullet \bar{B} \bullet \bar{A} + (C + C_R \bullet V_1) \bullet \bar{A} \bullet V_1 + B_R \bullet S \bullet V_1$$

(3) Incorporating the ON-condition (II) the equation becomes

$$V_{1_{(3)}} = (C + C_R \bullet V_1) \bullet \bar{B} \bullet \bar{A} + (C + C_R \bullet V_1) \bullet \bar{A} \bullet V_1 + C \bullet B + B_R \bullet S \bullet V_1$$

(4) The OFF-condition (II)(a) is equivalent to OFF-condition (I)(a)

$$V_{1_{(4)}} = (C + C_R \bullet V_1) \bullet \bar{B} \bullet \bar{A} + (C + C_R \bullet V_1) \bullet \bar{A} \bullet V_1 + B_R \bullet S \bullet V_1$$

(5) Incorporating the ON-condition (II)(b), the following results

$$V_{1_{(5)}} = (C + C_R \cdot V_1) \cdot \bar{B} \bullet \bar{A} + (C + C_R \bullet V_1) \cdot \bar{A} \cdot V_1 + C \bullet B + B_R \bullet S \cdot V_1 + (C + C_R \cdot V_1) \cdot \bar{B} \bullet \overline{(A + A_R \bullet V_5)}$$

Inspection of the last expression for V_1 shows that the last term in this equation $(C + C_R \bullet V_1) \bullet \bar{B} \bullet \overline{(A + A_R \bullet V_5)}$ functionally includes the first term of this equation $(C + C_R \bullet V_1) \bullet \bar{B} \bullet \bar{A}$, thus this term can be omitted.

Simplifying and rearranging, using Boolean algebra rules, leads to the following equation:

$$V_{1_{(5)}} = (C \cdot B) + (B_R \cdot S \cdot V_1) + (C + C_R \cdot V_1) \cdot \bar{A} \cdot V_1 + \bar{B} \cdot \overline{(A + A_R \cdot V_5)}$$

(6) The "ON-condition (III)" for simultaneous firing of two back-to-back engines is obtained by means of a pulse signal G*.

$$G^*_1 = \left[(\bar{D}_R \bullet B) + (\bar{C}_R \bullet A) \right]^*$$

The pulse signal is obtained by means of a "ONE SHOT" device that holds the pulse signal in the 1-state for a preselected time upon the appropriate triggering.

In addition to the pulse signal (G*), a back-to-back operation check signal, Z_{BB}, is obtained in the following manner:

$$Z_{BB} = V_1 \bullet V_{11} + V_5 \bullet V_7 + Z_{BB} \bullet (V_1 + V_5 + V_7 + V_{11})$$

Z_{BB} is a logic signal assuming the value O or 1 such that $Z_{BB} = 1$ if "ON-condition (III)" has occurred, and Z_{BB} will hold this value until V-signal termination. The purpose of Z_{BB} is to ensure selection of the proper turn off condition for the DBB mode and for the uncoupled yaw correction mode. If Z_{BB} goes high, the following DBB turn off condition is enabled (thruster number 1).

$$Z_{BB} \cdot V_1 \cdot (B + B_R \cdot \bar{A})$$

If Z_{BB} is equal to zero, the uncoupled yaw turn off condition is enabled for thruster number 1,

$$C_R \cdot \bar{Z}_{BB} \cdot V_1$$

(7) Incorporating the condition (III) into the command logic equations results in the following:

$$V_{1_{(6)}} = V_{1_{(5)}} + \bar{C} \cdot \bar{D} \cdot (G^*_1 + (B + B_R \cdot \bar{A}) \cdot V_1 \cdot Z_{BB}$$

(8) Incorporating the condition (IV) results in the following complete logic equation

$$\begin{aligned} V_1 = {} & B \cdot (C + Q \cdot (\bar{D}_R + V_1)) + (B_R \cdot S \cdot V_1) + (C + C_R \cdot V_1 \cdot \bar{Z}_{BB}) \\ & \cdot \left(\bar{A} \cdot V_1 + \bar{B} \cdot (\overline{A + A_R \cdot V_5}) \right) + \bar{C} \cdot \bar{D} \cdot \bar{Q} \cdot (G^*_1 \\ & + (B + B_R \cdot \bar{A}) \cdot Z_{BB} \cdot V_1) \end{aligned}$$

Figure 14 presents the complete set of the Boolean control equations for the DBB-DYT-DOTC concept.

3.2 Linear Signal Mixing

Linear Signal Mixing (LSM) provides a simple but effective means of controlling a multi-axis system of ON-OFF thrusters in which each thruster produces moments about two (or more) body axes, as do the roll and yaw thrusters of the six-engine arrangement shown in Figure 1. The relationship between LSM, conventional ON-OFF control and linear control, and a brief summary of LSM advantages and disadvantages is given below. A more detailed treatment is given in Reference 1.

In any system of thrusters where each thruster produces a moment about more than one axis, provisions must be made for "mixing" the errors sensed in the body axes and actuating the appropriate thrusters. In a linear system using proportional thrusters this mixing is straightforward; the error in a given axis is used to command a combination

$$G^* = (B \cdot \bar{D}_R + A \cdot \bar{C}_R)$$

$$H^* = (B \cdot \bar{C}_R + A \cdot \bar{D}_R)$$

$$\begin{aligned} V_1 = {} & B \cdot (C + Q \cdot (\bar{D}_R + V_1)) + (C + C_R \cdot \bar{Z}_{BB} \cdot V_1) \cdot \left[(\bar{A} \cdot V_1) + \bar{B} \cdot (\overline{A + A_R \cdot V_5}) \right] \\ & + (\bar{C} \cdot \bar{D} \cdot \bar{Q}) \cdot \left[G^* + Z_{BB} \cdot V_1 \cdot (B + B_R \cdot \bar{A}) \right] + (B_R \cdot S \cdot V_1) \end{aligned}$$

$$\begin{aligned} V_5 = {} & A \cdot (C + Q \cdot (\bar{D}_R + V_5)) + (C + C_R \cdot \bar{Z}_{BB} \cdot V_5) \cdot \left[(\bar{B} \cdot V_5) + \bar{A} \cdot (\overline{B + B_R \cdot V_1}) \right] \\ & + (\bar{C} \cdot \bar{D} \cdot \bar{Q}) \cdot \left[H^* + Z_{BB} \cdot V_5 \cdot (A + A_R \cdot \bar{B}) \right] + A_R \cdot \bar{S} \cdot V_5 \end{aligned}$$

$$\begin{aligned} V_7 = {} & B \cdot (D + Q \cdot (\bar{C}_R + V_7)) + (D + D_R \cdot \bar{Z}_{BB} \cdot V_7) \cdot \left[(\bar{A} \cdot V_7) + \bar{B} \cdot (\overline{A + A_R \cdot V_{11}}) \right] \\ & + (\bar{C} \cdot \bar{D} \cdot \bar{Q}) \cdot \left[H^* + Z_{BB} \cdot V_7 \cdot (B + B_R \cdot \bar{A}) \right] + (B_R \cdot S \cdot V_7) \end{aligned}$$

$$\begin{aligned} V_{11} = {} & A \cdot (D + Q \cdot (\bar{C}_R + V_{11})) + (D + D_R \cdot \bar{Z}_{BB} \cdot V_{11}) \cdot \left[(\bar{B} \cdot V_{11}) + \bar{A} \cdot (\overline{B + B_R \cdot V_7}) \right] \\ & + (\bar{C} \cdot \bar{D} \cdot \bar{Q}) \cdot \left[G^* + Z_{BB} \cdot V_{11} \cdot (A + A_R \cdot \bar{B}) \right] + (A_R \cdot \bar{S} \cdot V_{11}) \end{aligned}$$

*Signal is pulsed output from a one shot.

Figure 14. DOTC-DBB-DYT Control Logic Equations

of thrusters which produce pure moment about the desired axis. The thrust of each engine is then the algebraic sum of the commands received from each axis.

Figure 15a shows the mixing which would be appropriate for the system presented in Figure 1, if proportional thrusters operating about an average thrust level were used. (This would be inefficient in this application but it represents one possibility.) It may be seen that a positive yaw error would cause thrusters 1 and 5 to increase, and 7 and 11 to decrease. A simultaneous positive roll error would augment the yaw effects at thrusters 5 and 7 and oppose them at thrusters 1 and 11.

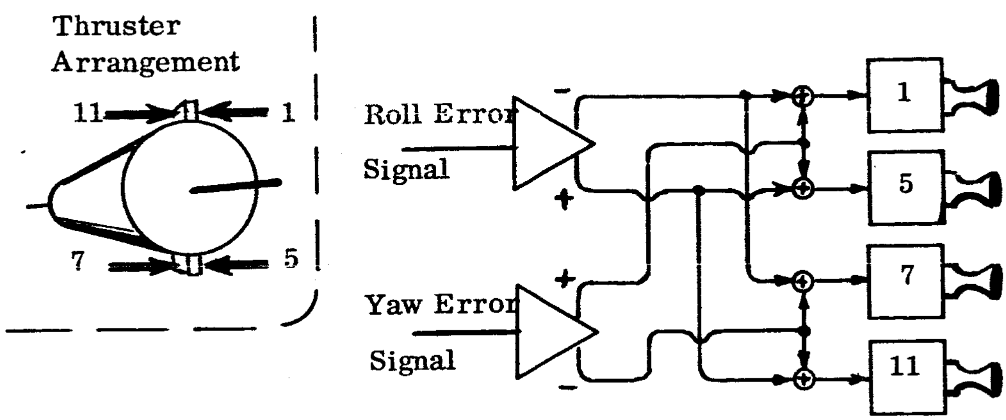
Where ON-OFF thrusters are used, some form of threshold logic must be employed. The conventional approach involves firing a pair of thrusters, chosen so that their moments add in the desired axis and cancel in the other, when the error in either axis exceeds some threshold value. Figure 15b illustrates that this system can be implemented simply by inserting threshold detectors in the error channels of the proportional system. The adders of the proportional system must now be taken as OR gates. Because mixing is accomplished downstream of the thresholds on a binary basis, this approach amounts to logic level signal mixing rather than linear signal mixing.

From Figure 15b, it can be seen that combined simultaneous roll and yaw errors large enough to exceed the thresholds will call for three thrusters to fire. This inevitably results in two thrusters firing back-to-back to no avail, with only the third thruster contributing a useful output. Fairly simple logic could be added to inhibit a thruster whenever its opposite member is commanded to fire and thus avoid simultaneous back-to-back firing. However, combined disturbances in roll and yaw could still cause firings of, say, thrusters 1 and 5 to alternate with firings of 5 and 11, resulting in a similar inefficiency because 1 and 11 firings cancel each other on the average.

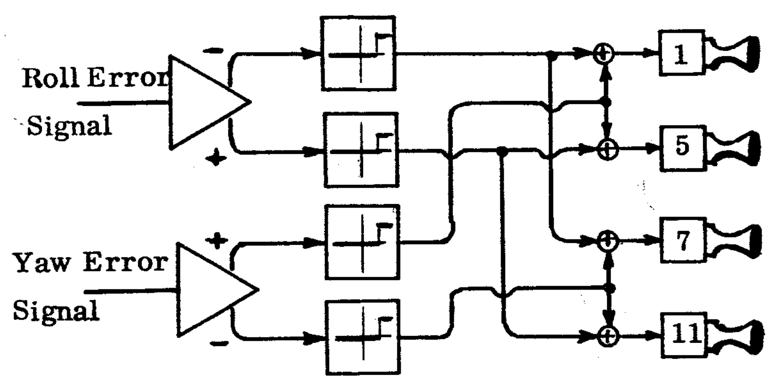
LSM is based on firing a thruster whenever the combined commands sent to it from the roll and yaw axes exceed a threshold level. Its implementation is derived from the linear system of Figure 15a by inserting threshold detectors in the signal to each thruster, as shown in Figure 15c. A more detailed block diagram of the complete LSM roll-yaw control loop is shown in Figure 16. The threshold and adder circuitry there has been combined and rearranged, but is functionally identical to that of Figure 15c.

LSM involves summing and differencing the roll and yaw errors, and using each combination to control a pair of back-to-back thrusters. In this way, LSM eliminates even the tendency for back-to-back firing; when the threshold controlling any thruster is exceeded, the threshold controlling its opposite member cannot be approached.

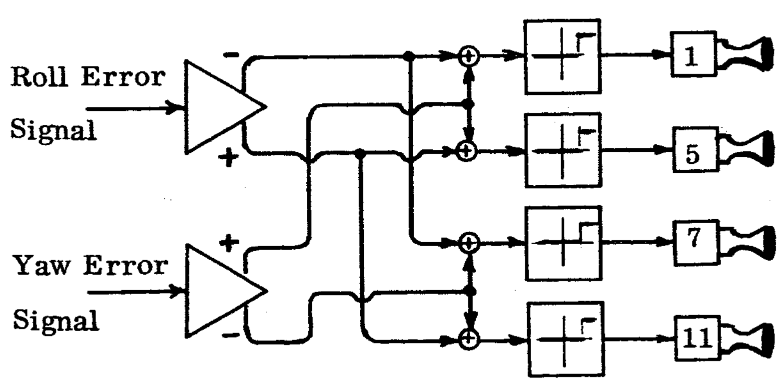
LSM tends to accomplish stabilization and control by firing thrusters singly, rather than in pairs, thereby improving efficiency. As may be seen in Figure 4, whenever the roll or yaw error approaches its threshold value, the threshold controlling one or the other of the two thrusters which tend to correct the error will be exceeded first, unless the error in the other axis is exactly zero. With proper starting conditions, it is theoretically possible to sustain a limit cycle indefinitely by alternately firing only thrusters



a) Proportional System Signal Mixing



b) Logic Level Signal Mixing



c) Linear Signal Mixing

Figure 15. Various Approaches to Control Signal Mixing with Coupled Thrusters

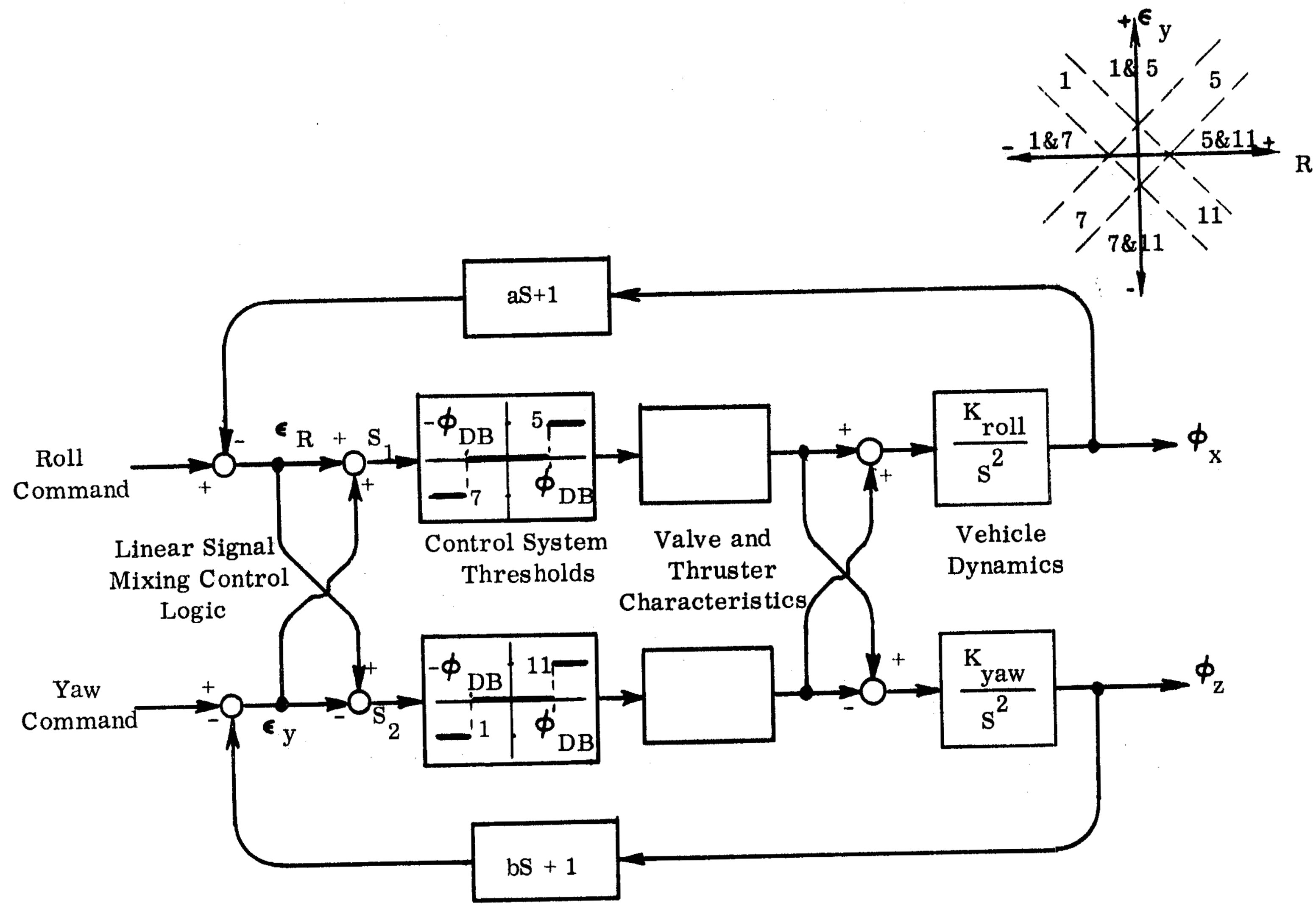


Figure 16. Linear Signal Mixing Block Diagram.

5 and 7 or 1 and 11. This situation represents the "best case" in terms of propellant consumption and number of thruster actuations per unit time. A worst case exists when the error in the axis of high inertia (yaw) is near its threshold. Small errors of either polarity in the other axis (roll) then cause a thruster firing which because of the low inertia and small error magnitude. quickly reverses the error and results in another firing, etc. The resultant rapid sequence of short firings will slowly give way to less frequent firings of larger duration as the combined effect of all firings eventually drives the yaw error away from the thresholds.

While LSM tends to correct small errors by firing thrusters singly, larger errors in either axis will cause two thrusters to fire despite small errors in the other axis. As a result, fast initial response to commands or large transient disturbances is provided. It may be seen from the firing diagram of Figure 4 that large errors will initially be corrected with two thrusters except when the roll and yaw errors are of nearly equal magnitude. A conventional control system, on the other hand, would fire two thrusters only when the error in one of the axes is within its threshold.

4.0 ANALYTICAL EVALUATION AND COMPARISON OF LOGIC CONCEPTS

4.1 GENERAL

The undisturbed limit cycle performance of the basic DOTC logic concepts were analytically evaluated and compared with that of more conventional control concepts. These studies are summarized in this section. The results are sufficiently valid to show clear trends and to demonstrate the feasibility of the DOTC concept, even though several simplifying assumptions, such as considering only best and worst cases were made. (More thorough evaluations resulting from computer studies, are given in Section 5.0.)

The fundamental assumption underlying these analyses is that a minimum firing time constraint, t_{min}, is imposed on each individual thruster, and a minimum differential firing time constraint, Δt, is imposed on each pair of thrusters fired differentially. These assumptions are reasonable. Single-shot flip-flops are used to provide an intentional t_{min} in some present day control systems to assure time for the multiple propellant valves to fully open, to prevent rapid chattering of threshold controlled systems under sustained disturbance situations and to achieve reasonable values of pulse specific impulse. A minimum Δt value equal to the turn-off delay of a thruster assembly automatically results when a closed loop method is used to control DOTC firing times. This is because control action in the roll axis does not start, and hence the second engine cannot be commanded off, until the thrust of the first engine begins to decay. The off command of the two thrusters will thus be separated in time by an amount not less than the time between the off-command of the first thruster and the start of its thrust decay; this time is the turn-off delay.

While the analytical evaluation requires that fixed and repeatable values for t_{min} and Δt be assumed, it is not necessary that specific values be known for them. Instead, these times and their ratio may be treated parametrically.

A single axis analytical evaluation and comparison of the DOTC-DBB logic concept and a conventional control system is given below. Following this is a dual axis comparison of DOTC-DBB, DOTC-DYT, LSM, and a conventional two-axis control system.

4.2 SINGLE AXIS COMPARISON OF DOTC-DBB AND A CONVENTIONAL SYSTEM

The following comparison demonstrates the feasibility of differential ON-time control and establishes the conditions under which it is superior to a conventional threshold type of ON-OFF control system in terms of propellant consumption and

Item	DOTC-DBB	Conventional
1. ON-time per cycle	$2(2t_{min} + \Delta t)$ (One thruster fires for t_{min} and the other for $t_{min} + \Delta t$ each half cycle)	$2t_{min}$ (One thruster fires for t_{min} each half cycle)
2. Thruster Actuations per cycle	4	2
3. OFF-time per cycle	$\dfrac{8\ \phi_{DB}}{\ddot{\phi}\Delta t}$ (vehicle coasts from one threshold, ϕ_{DB}, to the other at a rate equal to half the rate bit-$\ddot{\phi}\Delta t$-and back at an equal and opposite rate)	$\dfrac{8\ \phi_{DB}}{\ddot{\phi}\ t_{min}}$ (Same as DOTC, except that rate bit is $\ddot{\phi}\ t_{min}$)
4. ON-time per second (Item 1 ÷ Item 3) (ON-time Ratio)	$\dfrac{(2t_{min} + \Delta t)\ \Delta t\ \ddot{\phi}}{4\ \phi_{DB}}$ (ON-time is negligible compared to OFF-time, so that period of oscillation is given by Item 3)	$\dfrac{(t_{min})^2\ddot{\phi}}{4\ \phi_{DB}}$ (Same as DOTC)
5. Actuations per second (Item 2 ÷ Item 3)	$\dfrac{\ddot{\phi}\Delta t}{2\ \phi_{DB}}$	$\dfrac{\ddot{\phi}\ t_{min}}{4\ \phi_{DB}}$
6. $\dfrac{\text{DOTC ON-time Ratio}}{\text{Conv ON-time Ratio}}$	$\dfrac{(2t_{min} + \Delta t)\ \Delta t}{(t_{min})^2} = 2\left(\dfrac{\Delta t}{t_{min}}\right) + \left(\dfrac{\Delta t}{t_{min}}\right)^2$ (From Item 4)	
7. $\dfrac{\text{DOTC Actuations}}{\text{Conv Actuations}}$	$2\ \dfrac{\Delta t}{t_{min}}$ (From Item 5)	

Figure 17. Comparison of DOTC-DBB and Conventional Single Axis System

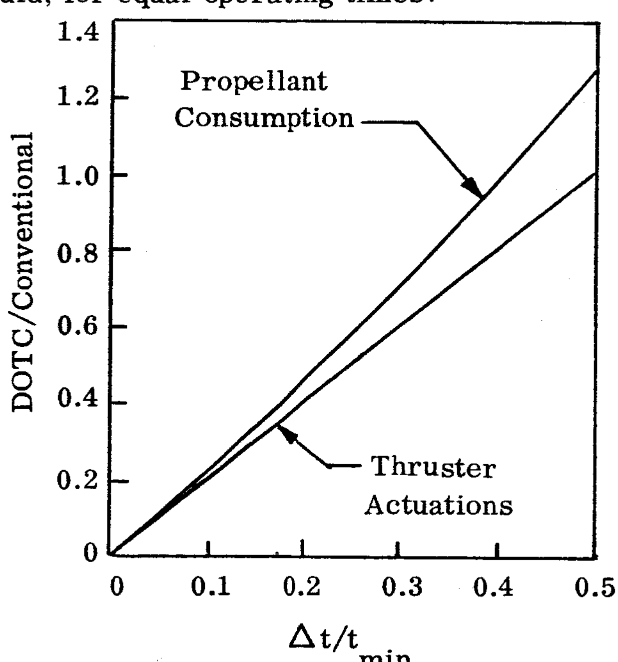
number of thruster actuations in operation. Only the differential back to back concept is considered, because other forms of DOTC (as well as Linear Signal Mixing) are inherently dual-axis concepts.

Expressions for thruster ON-times per second and number of actuations per second are derived in the table of Figure 17. From Item 6 of the figure, the ratio of DOTC ON-time per second to that of a conventional system is equal to $2(\Delta t/t_{min}) + (\Delta t/t_{min})^2$. This ratio is unity for $\Delta t/t_{min} = (\sqrt{2} - 1)$ or 0.414, and is favorable to DOTC for all lower ratios.

The ratio of ON-times per second is a good measure of the relative rates of propellant consumption between two systems if the specific impulses achieved for all thruster firings are equal. The specific impulse for short pulses actually varies somewhat with pulse duration, but in this comparison all pulses are of either t_{min} or $t_{min} + \Delta t$ duration and are thus nearly equal, particularly where $\Delta t \ll t_{min}$, as is required for DOTC to show a significant advantage. To whatever extent differences in specific impulse would influence the comparisons, they would do so to the advantage of DOTC, since all of its firings are as long or longer than those of the conventional system and hence achieve as high or higher values of specific impulse.

From Item (7) of Figure 17 it may be seen that the ratio of the number of DOTC thruster actuations to that of the conventional system is given by $2\,\Delta t/t_{min}$, indicating that the DOTC system requires fewer actuations per unit of time than a conventional system whenever $\Delta t/t_{min} < 0.5$.

The trend of relative propellant consumption and relative number of actuations as functions of $\Delta t/t_{min}$ is shown in the following sketch. Typical values for Δt and t_{min} are 5 and 50 milliseconds, respectively. Under these conditions a DOTC system would require only 21% of the propellant and 20% of the thruster actuations that a conventional system would, for equal operating times.



4.3 DUAL-AXIS EVALUATION AND COMPARISON OF LOGIC CONCEPTS

In order to demonstrate the potential propellant savings which can result from the application of Differential ON-Time Control concepts to a dual axis control system, an evaluation and comparison was made of the undisturbed limit cycle performances of dual axis systems using:

(1) Differential ON-Time Control with Differential Back-to-Back Firing (DOTC-DBB)

(2) Differential ON-Time Control with Differential Yaw Thrusting (DOTC-DYT)

(3) Linear Signal Mixing (LSM)

(4) A Conventional Uncoupled Control Concept (UCC)

The last system is essentially two single-axis control systems operating independently, each of which fires thrusters in pairs to avoid control coupling between axes.

4.3.1 Analytical Results

The system performance for these four control concepts was evaluated in terms of the ratios of total thruster ON-time and total number of thruster actuations to total thruster OFF-time. As discussed for the preceding single axis cases, the first ratio is a good relative measure of the rate of propellant consumption. Similarly, the second ratio is a good measure of thruster actuation rate. The performances of the various systems were normalized with respect to LSM for comparison.

The various expressions, results, and sub-results of this comparison are presented in Figure 18. The derivation of the items in the figure is discussed at the end of this section. The relative performance of each system, as shown in the last two columns, is a function of only $\Delta t/t_{min}$, previously discussed, and $\ddot{\phi}_x/\ddot{\phi}_z$, the ratio of roll control acceleration per engine to yaw acceleration per engine. The relative performances are plotted as functions of these parameters in Figures 19 and 20. It may be seen that the UCC system consumes four times the propellant as LSM, for all values of $\ddot{\phi}_x/\ddot{\phi}_z$, while all of the DOTC systems show a propellant advantage over LSM for control acceleration ratios greater than ten, and one DOTC system shows an advantage for control power ratios as low as 4. The DOTC-DBB logic generally shows an advantage over DOTC-DYT. The comparison in terms of thruster actuation rates is very similar to that for propellant consumption.

Typical values of the parameters of interest for the baseline vehicle are

$$\ddot{\phi}_x = 1 \text{ deg/sec}^2$$
$$\ddot{\phi}_z = 0.02 \text{ deg/sec}^2$$

		Limit Cycle Rate $\lvert\dot{\phi}_{L.C.}\rvert$	No. Thruster Actuations Per Cycle	ON-Time Per Cycle	OFF-Time Per Cycle Approximate	No. Thruste[rs] Per Unit [T...] (Apprx...)
UCC	Yaw	$\ddot{\phi}_z t_{min}$	4	$4 t_{min}$	$\frac{4\phi_{DB}}{\ddot{\phi}_z t_{min}}$	$\frac{(\ddot{\phi}_x + \ddot{\phi} \ldots}{\phi \ldots}$
	Roll	$\ddot{\phi}_x t_{min}$			$\frac{4\phi_{DB}}{\ddot{\phi}_x t_{min}}$	
LSM	Yaw	$\frac{1}{2} \ddot{\phi}_z t_{min}$	2	$2 t_{min}$	$\frac{8\phi_{DB}}{(\ddot{\phi}_x + \ddot{\phi}_z) t_{min}}$	$\frac{(\ddot{\phi}_x + \ddot{\phi} \ldots}{4 \phi \ldots}$
	Roll	$\frac{1}{2} \ddot{\phi}_x t_{min}$				
DOTC - DYT $\frac{t_{min}}{\Delta t} < \frac{1}{2}\left(\frac{\ddot{\phi}_x}{\ddot{\phi}_z}\right)$	Yaw	$\ddot{\phi}_z t_{min}$	8	$4\left(t_{min} + \frac{\Delta t}{2}\right)$	$\frac{8\phi_{DB}}{\ddot{\phi}_x \Delta t}$	$\frac{\ddot{\phi}_x \ldots}{\phi_{D} \ldots}$
	Roll	$\frac{1}{2} \ddot{\phi}_x \Delta t$		$2(2 t_{min} + \Delta t)$		
DOTC - DYT $\frac{t_{min}}{\Delta t} > \frac{1}{2}\left(\frac{\ddot{\phi}_x}{\ddot{\phi}_z}\right)$	Yaw	$\left(\ddot{\phi}_z t_{min}\right)_{Appr.}$	4	$4 t_{min}$	$\frac{8\phi_{DB}}{\ddot{\phi}_z (2 t_{min} - \Delta t)}$	$\frac{(\ddot{\phi}_x \Delta t \ldots}{\phi_{D} \ldots}$
	Roll	$\frac{1}{2} \ddot{\phi}_x \Delta t$	8	$2(4 t_{min} + \Delta t)$	$\frac{8\phi_{DB}}{\ddot{\phi}_x \Delta t}$	
DOTC - DBB $\frac{t_{min}}{\Delta t} > \frac{1}{2}\sqrt{\frac{\ddot{\phi}_x}{\ddot{\phi}_z}}$	Yaw	$\frac{1}{2} \ddot{\phi}_z \Delta t$	4	$2(2 t_{min} + \Delta t)$	$\frac{8\phi_{DB}}{\ddot{\phi}_x \Delta t}$	$\frac{\ddot{\phi}_x \ldots}{2\phi \ldots}$
	Roll	$\frac{1}{2} \ddot{\phi}_x \Delta t$				
DOTC - DBB $\frac{t_{min}}{\Delta t} > \frac{1}{2}\sqrt{\frac{\ddot{\phi}_x}{\ddot{\phi}_z}}$	Yaw	$\leq \ddot{\phi}_z t_{min}$	4	$2(2 t_{min})$	$\frac{4\phi_{DB}}{\ddot{\phi}_z t_{min}}$	$\frac{(\ddot{\phi}_x \Delta t + \ldots}{2\phi \ldots}$
	Roll	$\frac{1}{2} \ddot{\phi}_x \Delta t$		$2(2 t_{min} + \Delta t)$	$\frac{8\phi_{DB}}{\ddot{\phi}_x \Delta t}$	

Actuations ime)	ON-OFF-Time Ratio	Relative Actuation Rate Normalized With Respect to LSM	Relative ON-OFF-Time Ratio Normal With Respect to LSM
$z) t_{min}$ / DB	$\frac{(\ddot{\phi}_x + \ddot{\phi}_z) t_{min}^2}{\phi_{DB}}$	4	4
$z) t_{min}$ / DB	$\frac{(\ddot{\phi}_x + \ddot{\phi}_z) t_{min}^2}{4\phi_{DB}}$	1	1
t	$\frac{2\ddot{\phi}_x (2 t_{min} + \Delta t) \Delta t}{4\phi_{DB}}$	$\frac{4(\Delta t/t_{min})}{(1 + \ddot{\phi}_z/\ddot{\phi}_x)}$	$\frac{2(2 + \Delta t/t_{min})(\Delta t/t_{min})}{(1 + \ddot{\phi}_z/\ddot{\phi}_x)}$
$\ddot{\phi}_x t_{min})$ / 3	$\frac{\ddot{\phi}_x (6 t_{min} + \Delta t) \Delta t}{4\phi_{DB}}$	$\frac{4(\ddot{\phi}_z/\ddot{\phi}_x + \Delta t/t_{min})}{(1 + \ddot{\phi}_z/\ddot{\phi}_x)}$	$\frac{(6 + \Delta t/t_{min})(\Delta t/t_{min})}{(1 + \ddot{\phi}_z/\ddot{\phi}_x)}$
Δt / B	$\frac{\ddot{\phi}_x (2 t_{min} + \Delta t) \Delta t}{4\phi_{DB}}$	$\frac{2(\Delta t/t_{min})}{(1 + \ddot{\phi}_z/\ddot{\phi}_x)}$	$\frac{(2 + \Delta t/t_{min})(\Delta t/t_{min})}{(1 + \ddot{\phi}_z/\ddot{\phi}_x)}$
$\ddot{\phi}_z t_{min})$ / DB	$\frac{4\ddot{\phi}_z (t_{min})^2 + \ddot{\phi}_x (2t_{min} + \Delta t)\Delta t}{4\phi_{DB}}$	$\frac{2(2\ddot{\phi}_z/\ddot{\phi}_x + \Delta t/t_{min})}{(1 + \ddot{\phi}_z/\ddot{\phi}_x)}$	$\frac{4(\ddot{\phi}_z/\ddot{\phi}_x) + 2(\Delta t/t_{min}) + (\Delta t/t_{min})^2}{(1 + \ddot{\phi}_z/\ddot{\phi}_x)}$

Figure 18. Expressions Involved in the Comparison of Control Logic Concepts

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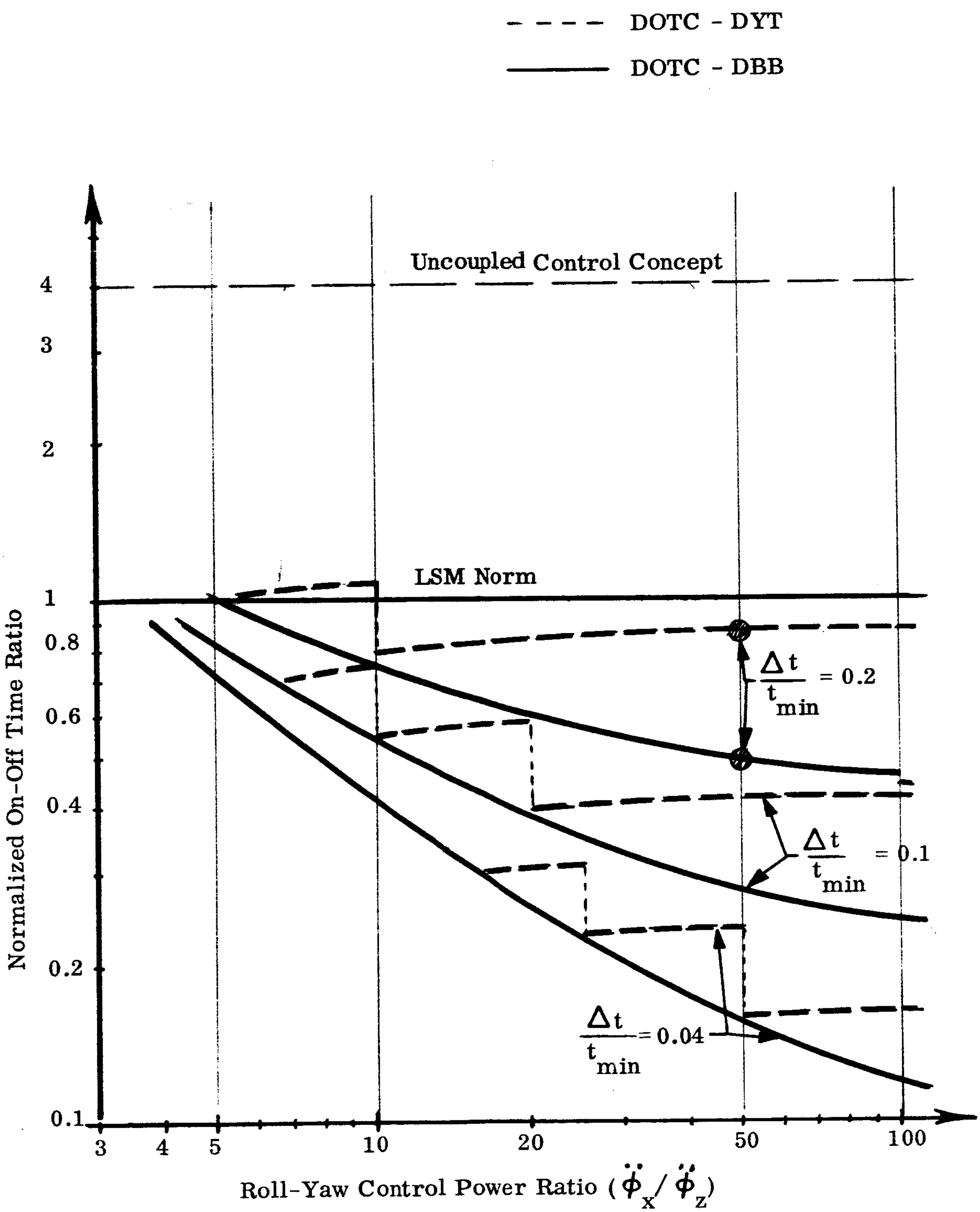


Figure 19. Relative Propellant Consumption of Four Basic Control Concepts

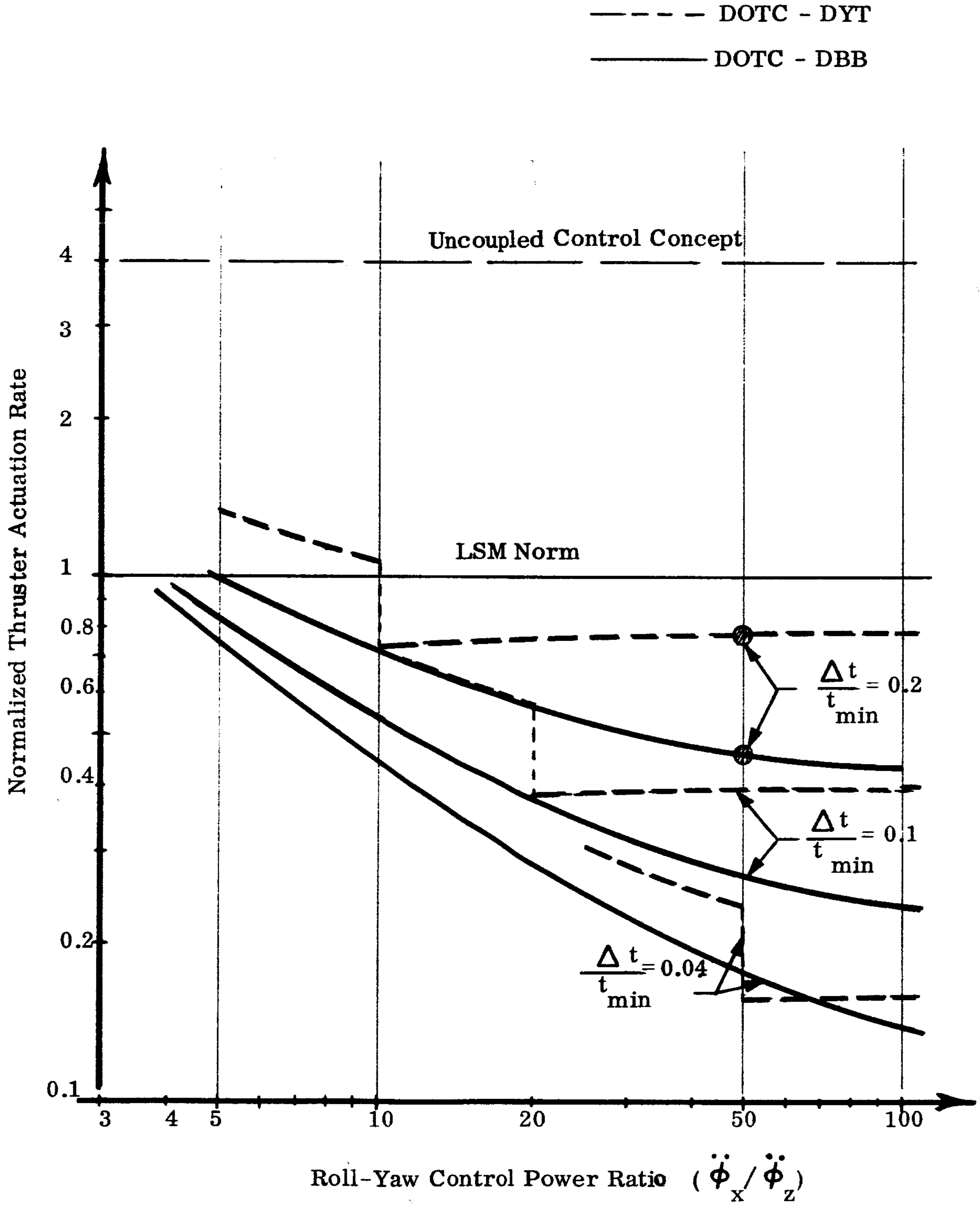


Figure 20. Relative Actuation Rates of Four Basic Control Concepts

$\Delta t = 0.010$ second

$t_{min} = 0.050$ second

Points corresponding to this set of values are shown on the two DOTC curves of Figures 19 and 20.

It can be seen that the DYT DOTC shows an advantage of about a 15% savings in fuel and number of thruster actuations over the LSM concept for these parameter values. However, the DBB concept shows a savings of about 50%.

It should be pointed out that the data shown for LSM represents a best case condition, in which limit cycle operation involves the firing of only two thrusters. Worst case LSM performance, discussed in Section 3.2 may be expected to be roughly comparable to that of the conventional Uncoupled Control Concept. On the other hand, it has been found that the DOTC system performance can be significantly improved over that shown here by combining the best features of DYT and DBB, and incorporating other modifications. This simplified analytical comparison does demonstrate the feasibility of DOTC. More complete comparisons of DOTC and LSM, as determined from computer simulations are given in the next section.

4.3.2 Description of Analytical Equations

The various items of Figure 18 were derived in the following manner:

(1) The first major row (including both roll and yaw) pertains to a system that uses UCC. The expressions for this simple case are self-explanatory, when it is kept in mind that $\ddot{\phi}$ is the acceleration per thruster, the engines are fired in pairs, the limit cycle rate is one-half of the minimum rate bit, and the OFF-time per cycle is the time to traverse from ϕ_{DB} to $-\phi_{DB}$ and back to ϕ_{DB}.

(2) The second major row of expressions corresponds to a system using LSM (refer to Section 3.2). For this case it was assumed that the dual axis system is in its slowest possible ("best") limit cycle; this corresponds to the situation illustrated in Figure 21 in the form of two phase plane diagrams showing the two limit cycle trajectories. It may be noted that only one thruster fires at a time and only two thrusters fire at all.

The limit cycle rates can be written as

$$(\dot{\phi}_x)_{LC} = \frac{1}{2}\ddot{\phi}_x t_{min} \text{ and } (\dot{\phi}_z)_{LC} = \frac{1}{2}\ddot{\phi}_z t_{min}.$$

The expression for the OFF-Time is derived from inspection of the trajectories of Figure 21. The drift time from one side of the trajectories to the other side is

$$1/2\left(\frac{t_{off}}{cycle}\right) \cong \frac{2\phi_{DB} - d_y}{\left(\frac{\ddot{\phi}_x t_{min}}{2}\right)} = \frac{d_y}{\left(\frac{\ddot{\phi}_z t_{min}}{2}\right)}$$

$$\frac{d_y}{\ddot{\phi}_z} + \frac{d_y}{\ddot{\phi}_x} = \frac{2\phi_{DB}}{\ddot{\phi}_x}$$

$$d_y = \frac{2\phi_{DB}\ \ddot{\phi}_z}{(\ddot{\phi}_x + \ddot{\phi}_z)}$$

$$\frac{t_{off}}{cycle} = 2\left(\frac{d_y}{\ddot{\phi}_z \frac{t_{min}}{2}}\right) = 2\left(\frac{4\phi_{DB}}{(\ddot{\phi}_x + \ddot{\phi}_z)\, t_{min}}\right)$$

The remaining computational steps for the LSM system are self-explanatory.

(3) The third major row in Figure 18 presents expressions valid for a system using DOTC-DYT. Figures 22a and 22b illustrate the limit cycling of this system by means of four phase plane diagrams. Two possible limit cycle modes can exist subject to the indicated condition.

Figure 22a illustrates an adverse case where the yaw attitude "hangs up" at one side of its threshold and limit cycles at double the roll axis limit cycle frequency. This phenomenon is due to the particular control strategy which requires corrective yaw thrusting when either threshold ($\pm\ \phi_{DB}$) in roll or yaw is exceeded (a more complete development of the DOTC-DYT concept is given in Section 3.1). From inspection of Figure 22a the following can be stated:

$$\frac{1/2\ (\text{Engine OFF-Time})}{\text{Cycle}} = \text{Drift Time across } 2\ \phi_{DB}$$

$$= \frac{2\phi_{DB}}{\ddot{\phi}_x \frac{\Delta t}{2}} = 1/2\left(\frac{8\phi_{DB}}{\ddot{\phi}_x \Delta t}\right)$$

$$\frac{\text{Engine ON-Time}}{\text{Cycle}} = 2\ (\text{Yaw-Roll thrust times}) + 2\ (\text{Yaw thrust times})$$

$$= 2\left[t_{min} + (t_{min} + \Delta t)\right] + 2\left[2\left(t_{min} + \frac{\Delta t}{2}\right)\right]$$

$$= 4\ (2t_{min} + \Delta t)$$

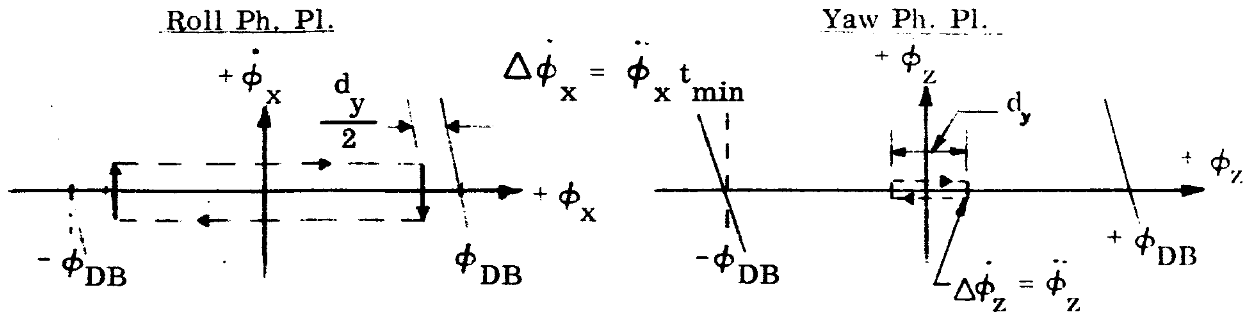
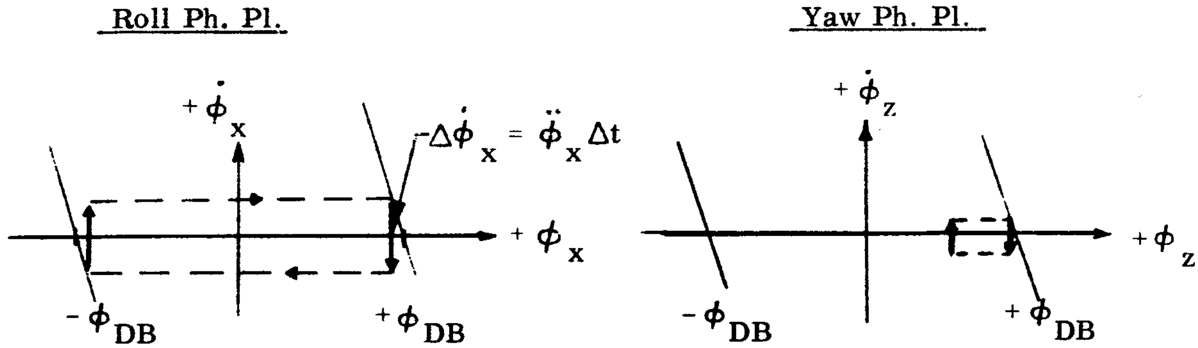


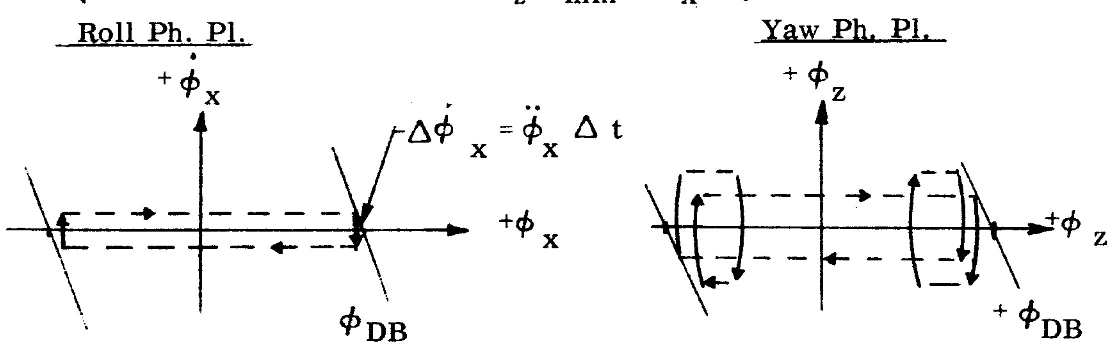
Figure 21. Undisturbed Lowest Order Limit Cycling of a Dual Axis System Employing LSM Control

(First Possible Mode - $\ddot{\phi}_z 2t_{min} < \ddot{\phi}_x \Delta t$)



a. Simple Limit Cyling of a System Using DOTC - DYT

(Second Possible Mode - $\ddot{\phi}_z 2t_{min} > \ddot{\phi}_x \Delta t$)



b. 1st Higher Order Limit Cycling of a System Using DOTC - DYT

Figure 22. Possible Limit Cycle Modes Using DOTC - DYT

The remaining computational steps to complete this particular row of Figure 18 are self-explanatory.

Figure 22b illustrates a more complex limit cycling that will occur if the yaw limit cycle drift rate (1/2 $\ddot{\phi}_z 2t_{min}$) is larger than the roll drift rate (1/2 $\ddot{\phi}_x \cdot \Delta t$). From inspection of Figure 22b, the following relationships can be formulated:

$$\frac{\text{OFF-Time}}{\text{Cycle}} = \frac{8\phi_{DB}}{\ddot{\phi}_x \Delta t} \qquad \text{(same as for the previous case)}$$

$$\begin{aligned}\text{Thruster } \frac{\text{ON-Time}}{\text{cycle}} &= 2 \text{ (yaw roll thrust times)} \\ &\quad + 4 \text{ (yaw thrust times)} \\ &= 2\left[t_{min} + (t_{min} + \Delta t)\right] + \left[4\,(2t_{min})\right] \\ &= 2\,(6\,t_{min} + \Delta t)\end{aligned}$$

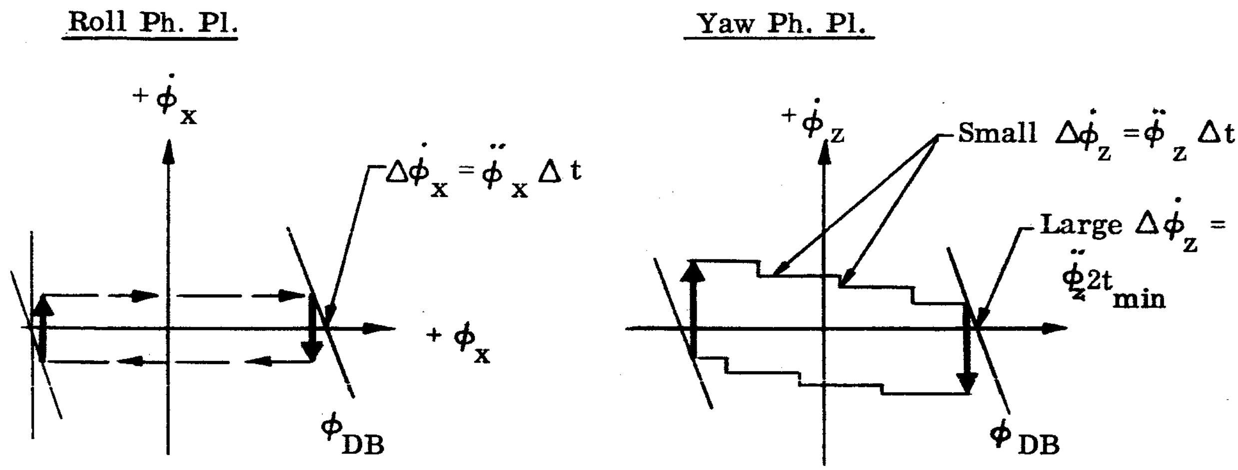
$$\text{ON-OFF Time Ratio} = \ddot{\phi}_x \frac{(6\,t_{min} + \Delta t)\,\Delta t}{4\phi_{DB}}$$

The remaining computations to complete this particular row of Figure 18 are self-explanatory.

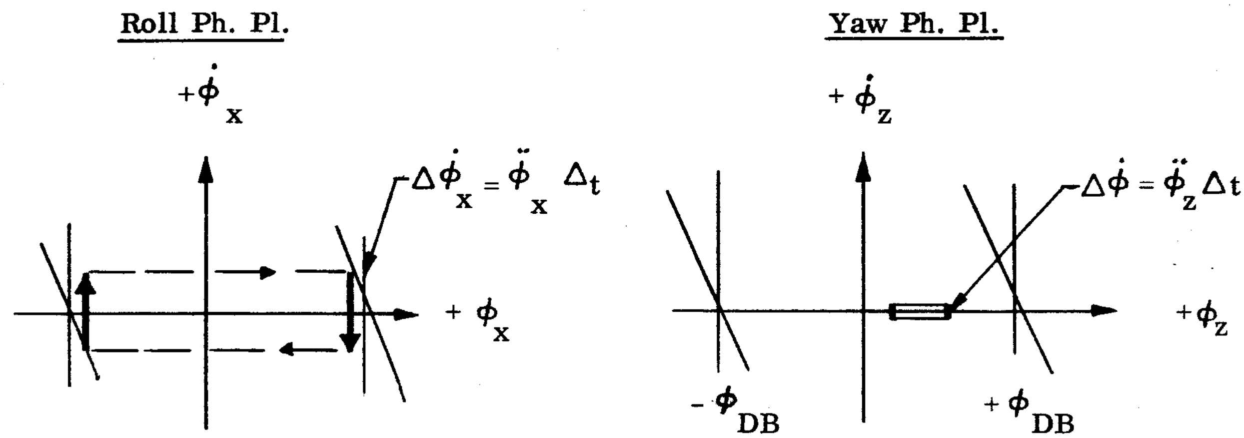
(4) The fourth and last major row of expressions in Figure 18 corresponds to a system that uses the DOTC-DBB concept. Figures 23a and 23b illustrate the two possible modes of limit cycling by means of four phase plane diagrams.

The particular limit cycling characteristics are due to the DOTC-DBB control strategy which requires that upon exceeding the roll threshold, back to back thrusting will occur with differential shutoff such that the roll rate is reversed and the yaw rate is reduced. A more complete development of this particular DOTC-DBB concept is given in Section 3.1

The period of the yaw limit cycle is estimated as follows: The portion of the yaw trajectory in Figure 23a between the two large corrective actions at the threshold ($\pm\phi_{DB}$) is characterized by some "average disturbance" acceleration defined as follows:



a. Limit Cycling of a System Using DOTC - DBB for $\frac{t_{min}}{\Delta t} > \frac{1}{2}\sqrt{\frac{\ddot{\phi}_x}{\ddot{\phi}_z}}$



b. Limit Cycling of a System Using DOTC - DBB for $\frac{t_{min}}{\Delta t} < \frac{1}{2}\sqrt{\frac{\ddot{\phi}_x}{\ddot{\phi}_z}}$

Figure 23. Possible Modes of Limit Cycling Using the DOTC - DBB Concept

$$\bar{\ddot{\phi}}_{xz} \triangleq \ddot{\phi}_z \quad \frac{2\Delta t}{P_x}$$

(where P_x is the roll axis limit cycle period)

$$P_x \cong \frac{8\,\phi_{DB}}{\ddot{\phi}_x \Delta t} \qquad \text{(obtained from inspection of Figure 23a)}$$

Figure 24 below presents the yaw limit cycle trajectory under the influence of $\bar{\ddot{\phi}}_{xz}$.

From inspection of Figure 24 and using the familiar parabolic trajectory relationships, it can be shown that

$$\underline{\text{Yaw:}} \quad \frac{\text{OFF-Time}}{\text{Cycle}} = \frac{4\,\phi_{DB}}{\ddot{\phi}_z\, t_{min}}$$

$$\frac{\text{ON-Time}}{\text{Cycle}} = 2\,(2\,t_{min})$$

$$\text{Also, if } \bar{\ddot{\phi}}_{xz} \geq 1/2 \quad \frac{(\ddot{\phi}_z\, 2t_{min})^2}{2\,\phi_{DB}} = \frac{(\ddot{\phi}_z\, t_{min})^2}{\phi_{DB}}$$

no separate yaw limit cycle will exist.

$$\frac{\ddot{\phi}_x\, \ddot{\phi}_z\, (\Delta t)^2}{4\phi_{DB}} \geq \frac{4(\ddot{\phi}_z)^2\ (t_{min})^2}{4\,\phi_{DB}}$$

which is rewritten as

$$1/2 \sqrt{\ddot{\phi}_x / \ddot{\phi}_z} \geq t_{min}/\Delta t$$

The remaining computational steps for completing the last two in Figure 18 are straightforward.

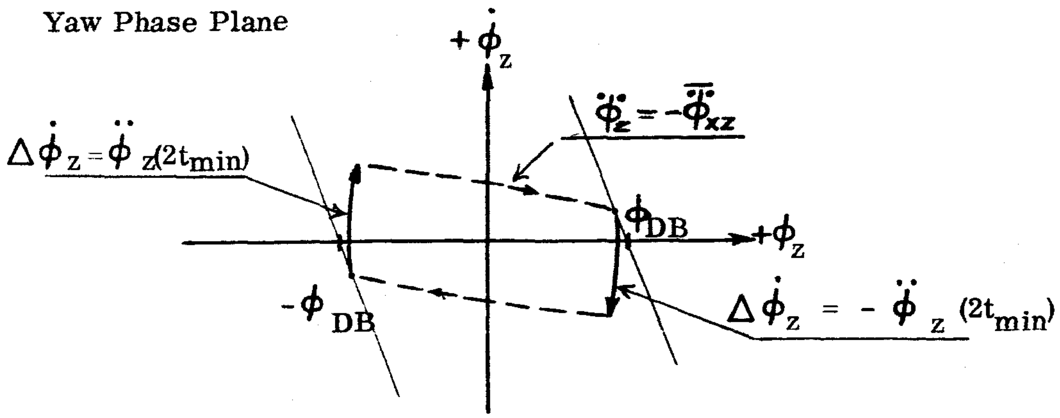


Figure 24. Approximate Representation of DOTC-DBB Yaw Limit Cycle

5.0 COMPUTER EVALUATION AND COMPARISON STUDIES

5.1 ANALOG SIMULATION STUDIES

An analog computer simulation incorporating digital logic circuitry was developed as part of the study program. This simulation was originally conceived as the primary means for evaluating DOTC concepts and comparing them with LSM. However, its complexity was such that it proved difficult to work with and difficult to modify to incorporate the more complex DOTC logics that were developed to overcome limitations of the earlier, simpler, approaches. Hence, a digital computer simulation, discussed in Part 5.2 of this section, was used to generate the majority of the final evaluation data. The analog simulation studies did, however, provide significant insight into the fundamental operating characteristics of DOTC systems. Specifically, they:

(1) Led to a modification of the initial analytical estimates of the performance of the DOTC-DYT-YP concept, which had failed to consider the "worst case" phasing of the roll and yaw limit cycles. (The comparisons of Section 4.0 reflect the modified estimates.)
(2) Indicated certain modifications of the original DOTC-DYT-YP logic which would improve its performance.
(3) Revealed the necessity for single-shot timing devices if minimum firing time restrictions were to be enforced.
(4) Indicated that approximating the pitch axis thruster disturbance coupling effects into roll and yaw by constant average torques would be valid and would reduce simulation complexity.

5.1.1 Simulation Description

The analog simulation comprised an outboard digital logic package to implement the DOTC logic and the time scale control, circuits to mechanize the turn-on and turn-off delays of four thrusters, four circuits to ensure a minimum thruster on-time, and parallel integration loops for computations in two different time scales. The pitch, roll and yaw degrees of freedom were simulated.

A diagram of the major blocks of analog elements is shown in Figure 25. Both LSM and the DOTC-DYT-YP were simulated.

The development of the analog simulation was not straightforward due to the impossibility of using a single time scale for this particular type of problem. The very short thruster on-times produce low vehicle angular rates which result in long "coast" times between thruster firings. For example, the time required for the vehicle error signal to drift across the dead band can be on the order of from 80 to 400 seconds, depending on the effective minimum rate bit obtained by the control logic. On the other

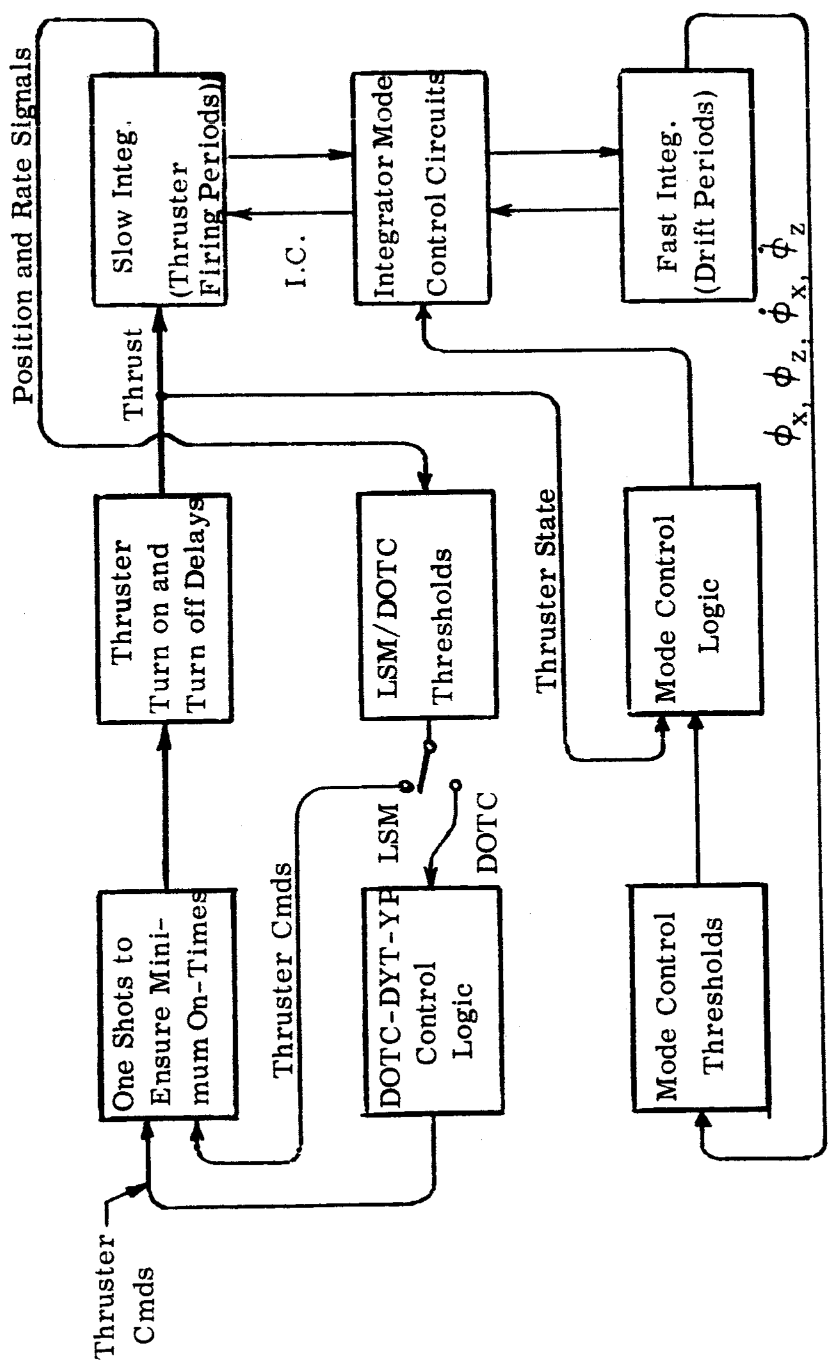


Figure 25. LSM/DOTC Analog Block Diagram

extreme. thruster firings can be accomplished in times of 50 ms with associated differential turn-off times on the order of 10 ms. It can be seen that the time scaling requirements for the different phases of this problem conflict greatly. The time scaling problem was solved by using two separate time scales with integrator mode control logic. The 50 ms thruster firings were scaled to require five seconds of computer time. The 100 second drift times were scaled to require four seconds.

5.1.2 Undisturbed Limit Cycle Characteristics of LSM and DOTC-DYT-YP

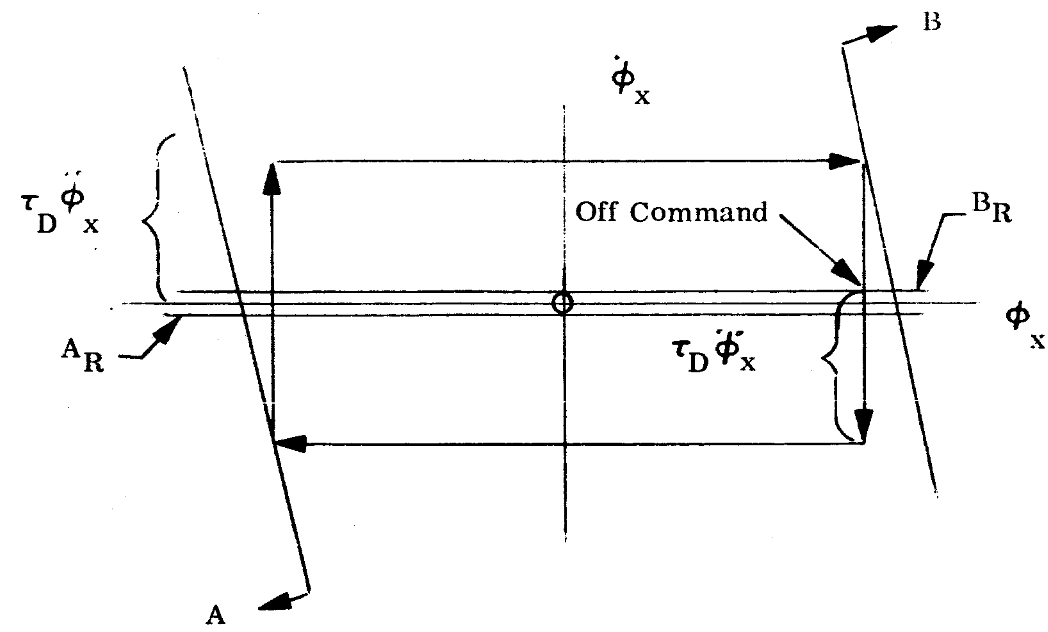
This DOTC concept provides for control of the roll axis by differential yaw thruster firings as described in Section 3.0 The DOTC logic shown in Figure 11 was investigated over a range of yaw control accelerations, engine turn-off delays, and rate threshold values. Yaw control acceleration values of 0.02, 0.05, and 0.1 deg/sec^2 were employed in the study of the undisturbed low rate limit cycle behavior of this DOTC concept. The following table shows the initial conditions which were used with each of the control acceleration combinations for all analog runs.

$\ddot{\phi}_x$	$\ddot{\phi}_z$	$\dot{\phi}_x$ (O)	ϕ_x (O)	$\dot{\phi}_z$ (O)	ϕ_z (O)
$\frac{\text{Deg}}{\text{Sec.}^2}$	$\frac{\text{Deg}}{\text{Sec.}^2}$	$\frac{\text{Deg}}{\text{Sec.}}$	Deg	$\frac{\text{Deg}}{\text{Sec.}}$	Deg
1	0.02	+0.04	0	+0.002	0
1	0.05	+0.04	0	+0.005	0
1	0.10	+0.04	0	+0.10	0

The indicated initial values of vehicle rates are in the range of those encountered during limit cycle operation and were chosen because it was desired primarily to study the limit cycle properties of the DOTC logic apart from consideration of problems of convergence from large rate or position errors. Roll and yaw error threshold values of ±one degree, a roll control acceleration of 1.0 deg/sec^2 per engine, and a minimum thruster on-time value of 50 ms were employed in all simulation studies.

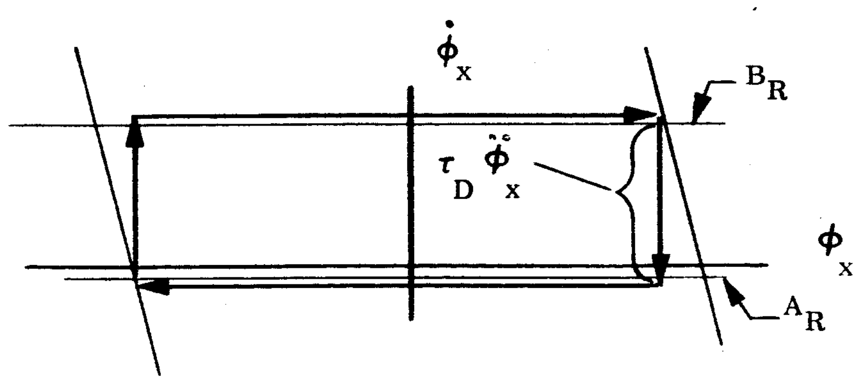
One of the characteristics of DOTC demonstrated by the simulator was that the A_R and B_R thresholds could be adjusted to develop a net roll axis rate bit equal to the thruster turn-off delay rather than twice that value, as occurs for settings of $A_R = B_R = 0$. It also showed that the roll axis limit cycle could be made asymmetric by proper choice of A_R and B_R threshold values.

The relation between the roll rate thresholds, the thruster turn-off delay time and the roll axis net angular rate bit is illustrated in the following roll axis phase plane diagram.



It can be seen that if the A_R and B_R thresholds are set to zero, the limit cycle rates will be $\pm (\tau_D \ddot{\phi}_x)$ and the net roll angular rate bit will be $(2 \tau_D \ddot{\phi}_x)$. The rate thresholds can be adjusted to slightly less than $\pm(0.5\, \tau_D \ddot{\phi}_x)$ and the net roll axis angular rate bit will then be only slightly larger than $(\tau_D \ddot{\phi}_x)$. It does not appear to be possible to adjust the rate thresholds to produce smaller rate bits (although smaller limit cycle rates in one direction can be achieved through asymmetry). Rate bits larger than $\tau_D \ddot{\phi}_x$ can be generated by locating A_R at any desired distance above the $\dot{\phi}_x = 0$ axis and locating B_R below it. This technique would be useful under large disturbance conditions.

By adjusting the rate threshold values to be unequal, the roll limit cycle can easily be made unsymmetrical. The following roll axis phase plane diagram illustrates threshold settings which produce rate bits nearly equal to $\tau_D \ddot{\phi}_x$ and also produces a limit cycle rate of almost zero, in one direction.



It can be seen that by proper choice of thresholds, one of the limit cycle rates can be made as near zero as desired (within the limitations of rate sensor accuracy and turn-off delay repeatability), and thus the limit cycle period can be made as long as desired.

It should be pointed out that other control logic concepts beside DOTC can be used to provide asymmetrical limit cycles (for example, see Reference 2). In order to avoid crediting the DOTC concept with attributes which are not unique to it, all performance comparisons in this study were made on the basis of symmetrical limit cycles. However, it is an advantage of DOTC that, unlike LSM, it inherently incorporates means for providing unsymmetrical limit cycles.

The influences of the rate threshold values and thruster turn-off delay on the net roll axis rate bit were demonstrated on the analog computer by simulating the conditions listed in the following table.

Thruster Turn Off Delay msec	A_R Threshold deg/sec	B_R Threshold deg/sec	Positive Roll Rate deg/sec	Negative Roll Rate deg/sec	Net Roll Rate Bit deg/sec
20	0	0	+0.02	-0.02	0.04
5	0	0	+0.005	-0.005	0.01
20	-0.01	+0.01	+0.01	-0.01	0.02
5	-0.00025	+0.00375	+0.00475	-0.00125	0.006

Figure 26 presents the relative engine on-time parameter versus yaw control acceleration per engine for various values of thruster turn-off delay and rate threshold level. It illustrates the manner in which the limit cycle fuel consumption is affected by: (1) the ability to control the roll axis rate bit with the control logic rate thresholds, (2) the effect of value of the thruster turn-off delay time, and (3) the asymmetry of the roll limit cycle which can be induced by choice of the rate threshold values.

Runs 5, 6, and 7 all employed thruster turn-off delays of 20 ms and A_R and B_R rate threshold values of zero. Thus, the differential on-time was 40 ms and the rate bit was 0.04 deg/sec. Runs 9, 10, and 11 were performed with turn-off delay values of five ms and rate threshold values of zero leading to 10 ms on-times and 0.01 deg/sec rate bits. The latter set of runs used approximately five to six times less propellant per unit time than the former.

The vehicle control acceleration and engine delay parameters were the same values for runs 5 and 8. However, the fuel consumption of run 8 is only about one half that of run 5 as a result of adjustment of the roll rate thresholds to reduce the roll axis limit cycle rate bit to 0.02 deg/sec instead of 0.04. Run 21 shows the manner in which fuel consumption was reduced to very small values by employing the thresholds to develop an asymmetric rate condition. The relative fuel consumption of run 21 is only about 25% of that obtained with $A_R = B_R = 0$ and it could be reduced still further.

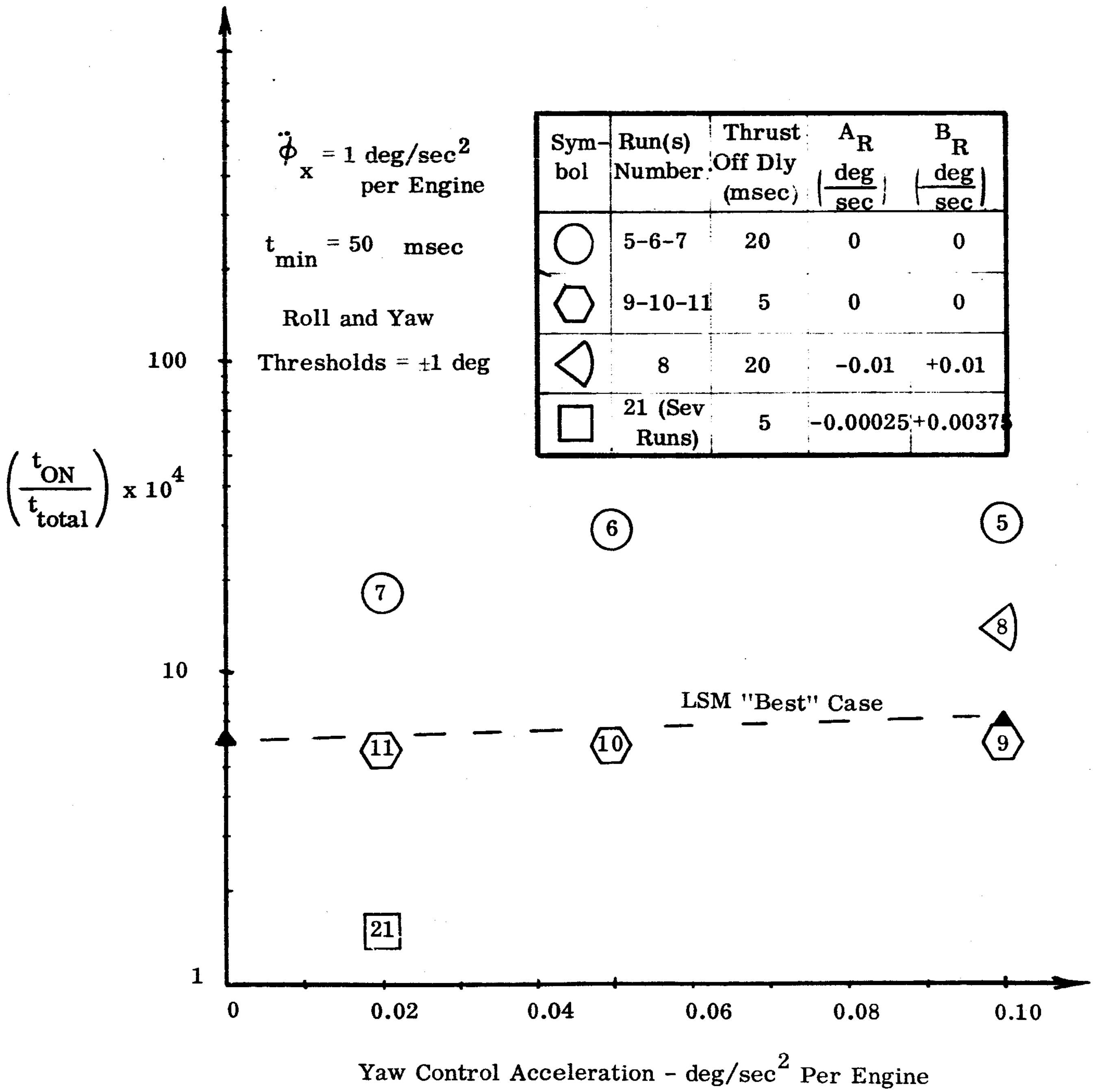


Figure 26. ON-Time Ratio of DOTC-DYT-YP for Various Values of System Parameters

The "best" case operation of LSM is also shown in Figure 26. It is seen that the undisturbed performance of the original DOTC-DYT-YP concept is clearly superior to LSM only if the rate thresholds are adjusted to produce on-times of 5 ms or unsymmetrical limit cycles.

5.1.3 Disturbance Effects

An investigation of the effects of roll axis disturbances was undertaken with the analog simulation. Although quantitative data was not obtained, (such data was later obtained from the digital simulation), the analog simulation did demonstrate that DOTC is not advantageous in the presence of significant roll disturbances. This results from the fact that DOTC, at least in its basic form, is primarily a means of achieving small angular rate bits, whereas disturbances are best controlled with the biggest bits that can be used while maintaining "one sided" operation such that all thruster firings oppose the disturbance. As mentioned previously, however, DOTC inherently provides means (the A_R and B_R thresholds) by which large bits of any size can be developed. An advanced DOTC concept which included means for determining the optimum bit size for any situation, would show an advantage over conventional systems with the same means, by virtue of its ability to achieve smaller rate bits when required.

This phase of the analog simulation study also demonstrated that the roll disturbances induced by pitch firings were, over a wide range of pitch disturbances and pitch control characteristics, of small enough amplitude and high enough frequency to be conveniently represented as constant disturbances.

5.2 DIGITAL COMPUTER SIMULATION RESULTS

Because the analog simulation proved less versatile and convenient to work with than anticipated, a digital computer simulation of the dual axis (yaw-roll axes) attitude control systems was programmed. The program was developed using a combination of FORTRAN IV and DSL/90 (both are IBM developed programming languages). The digital simulation included the same roll-yaw system characteristics that were included in the analog computer simulation.

The digital computer simulation was used to check out and evaluate the final combined DBB/DYT version of the DOTC logic described in Section 3.0 and presented in Figure 14. Simulation runs of systems using earlier and less complete versions of DOTC aided significantly in the development of the final version of the logic equations studied here. The digital computer simulation was used to demonstrate and evaluate the following effects.

(1) The influence of the initial conditions on the relative on-time convergence characteristics of LSM.

(2) Comparative evaluations of LSM and DOTC-DBB-DYT in the no disturbance situation.

(3) Comparative evaluation of LSM and DOTC-DBB-DYT in the presence of low level disturbances in roll and yaw.

(4) Comparative evaluations of LSM and DOTC-DBB-DYT in the presence of a large roll disturbance.

Figure 27 presents a summary of the pertinent parameters, initial conditions, and some results of the digital computer simulation runs.

5.2.1 Effects of Initial Conditions on the Limit Cycle Performance of a System Using LSM.

For a dual axis control system that employs LSM, there can exist several possible limit cycle operating modes. These steady state operating characteristics can vary with different initial conditions and (or) transient disturbance effects. In order to demonstrate this phenomenon, the digital computer simulation was used to perform convergence runs for two sets of initial conditions and two values of control power ratio (two values of yaw control power).

Parameter values and performance data for this set of runs, (1 through 4) are listed in figure 27.

These runs clearly demonstrate the significant effect that even a relatively small variation in the initial yaw rate has on the overall system performance. Figure 28 presents time histories for two characteristic system variables as obtained from digital simulation runs 1 and 2. One operating characteristic of significance is the number of thruster actuators

Turn-On Delay = 20 msec, Turn-Off Delay = 5 msec, t

Run No.	Type of Control Syst.	Control Accel. Roll $(\ddot{\phi}_x)_C$ deg/sec^2	Control Accel. Yaw $(\ddot{\phi}_z)_C$ deg/sec^2	Initial Conditions Roll $(\dot{\phi}_x)(O)$ deg/sec	Initial Conditions Roll $\phi_x(O)$ deg.	Initial Conditions Yaw $\dot{\phi}_z(O)$ deg/sec	Initial Conditions Yaw $\phi_z(O)$ deg	Disturbances Roll $(\ddot{\phi}_x)_D$ deg/sec^2	Disturbances Yaw $(\ddot{\phi}_z)_D$ deg/sec^2	Total Run Time Seconds	No. of T… No. Starts No.	Σ S
1	LSM	1.0	0.02	0.025	0.0	0.005	0.0	0.0	0.0	2000	11	
2	LSM		0.02			0.025				2000	22	
3	LSM		0.1			0.005				2000	7	
4	LSM		0.1			0.025				2000	12	
5	LSM	1.0	0.02	0.025	−0.125	0.001	0.94	0.0	0.0	2000	21	
6	DOTC							0.0		2000	3	
7	LSM							50×10^{-6}		2000	20	
8	DOTC							50×10^{-6}		1000	4	
9	LSM							100×10^{-6}		2000	21	
10	DOTC							100×10^{-6}		1000	7	
11	LSM							0.0	100×10^{-6}	2000	1	
12	DOTC								100×10^{-6}	2000	0	
13	LSM								200×10^{-6}	1500	0	
14	DOTC								200×10^{-6}	2000	0	
15	DOTC							0.0	$100 \times 10^{-6} \phi_z$	2000	0	
16	LSM							200×10^{-6}	0.0	2000	32	
17	DOTC							200×10^{-6}	0.0	2000	18	
18	DOTC*							200×10^{-6}	0.0	2000	17	
19	LSM							0.25	0.0	35	15	
20	DOTC							0.25	0.0	30	77	
21	DOTC*	1.0	0.02	0.025	−0.125	0.001	0.94	0.25	0.0	55	124	

* Modified DOTC-DBB-DYT, Back-to-Back Mode Blocked Out (Q = 1)

nin = 50 msec, $B_R = +10^{-4}$ deg/sec = - A_R

hruster Actuations and Accumulat. Burn Time							ON-time Ratio (Steady State)	No. Thruster Actuations Per Hour (Average Value)	Approx. Time to Reach (Steady State)	Approx. No. Yaw Limit Cycles During Simul. Run
1	No. 5		No. 7		No. 11					
t_{on}	Starts	$\sum t_{on}$	Starts	$\sum t_{on}$	Starts	$\sum t_{on}$				
econds	No.	Seconds	No.	Seconds	No.	Second	x 10^{-4}	No./Hour	Seconds	Cycles
).549	11	0.549	15	0.749	15	0.749	13	93	800	2-3/4
..098	20	1.001	34	1.7	36	1.796	28	202	1800	3-1/4
).349	5	0.25	6	0.3	7	0.37	6.4	45	1500	5
).609	9	0.4598	12	0.6153	15	0.7609	13	86	1000	8
..0824	22	1.1276	23	1.1597	22	1.1206	23	158	1000	3
).172	5	0.283	5	0.292	3	0.1504	4.5	29	2000	1/2
).999	18	0.896	19	0.948	18	0.898	19	135	1800	2-3/4
).2467	1	0.050	2	0.1319	5	0.2507	7	43	700	1/4
1.0912	20	1.002	24	1.1955	20	1.0416	22	153	1000	3
).4332	1	0.050	2	0.14031	8	0.4011	10	65	400	1/4
).05	1	0.05	102	5.09	101	5.04	51	369	1000	
)	0	0	91	5.044	91	5.0241	50	327	200	
)	0	0	155	7.752	156	7.7597	100	758	200	
)	0	0	180	10.026	180	10.006	100	649	100	
)	0	0	89	4.9964	89	4.9654	50	320	200	
1.60	29	1.45	33	1.65	27	1.35	30	218	100	
1.085	20	1.002	20	1.236	18	0.902	21	137	500	
0.897	13	0.653	17	0.879	14	0.721	16	110	1600	
4.942	0	0	14	3.894	0	0	2500+			
7.508	77	3.775	77	7.558	77	3.776	Approx 7500			
6.8717	0	0	124	6.919	0	0	2500+			

Figure 27. Parameter Values, Initial Conditions and Results of the LSM/DOTC Digital Studies

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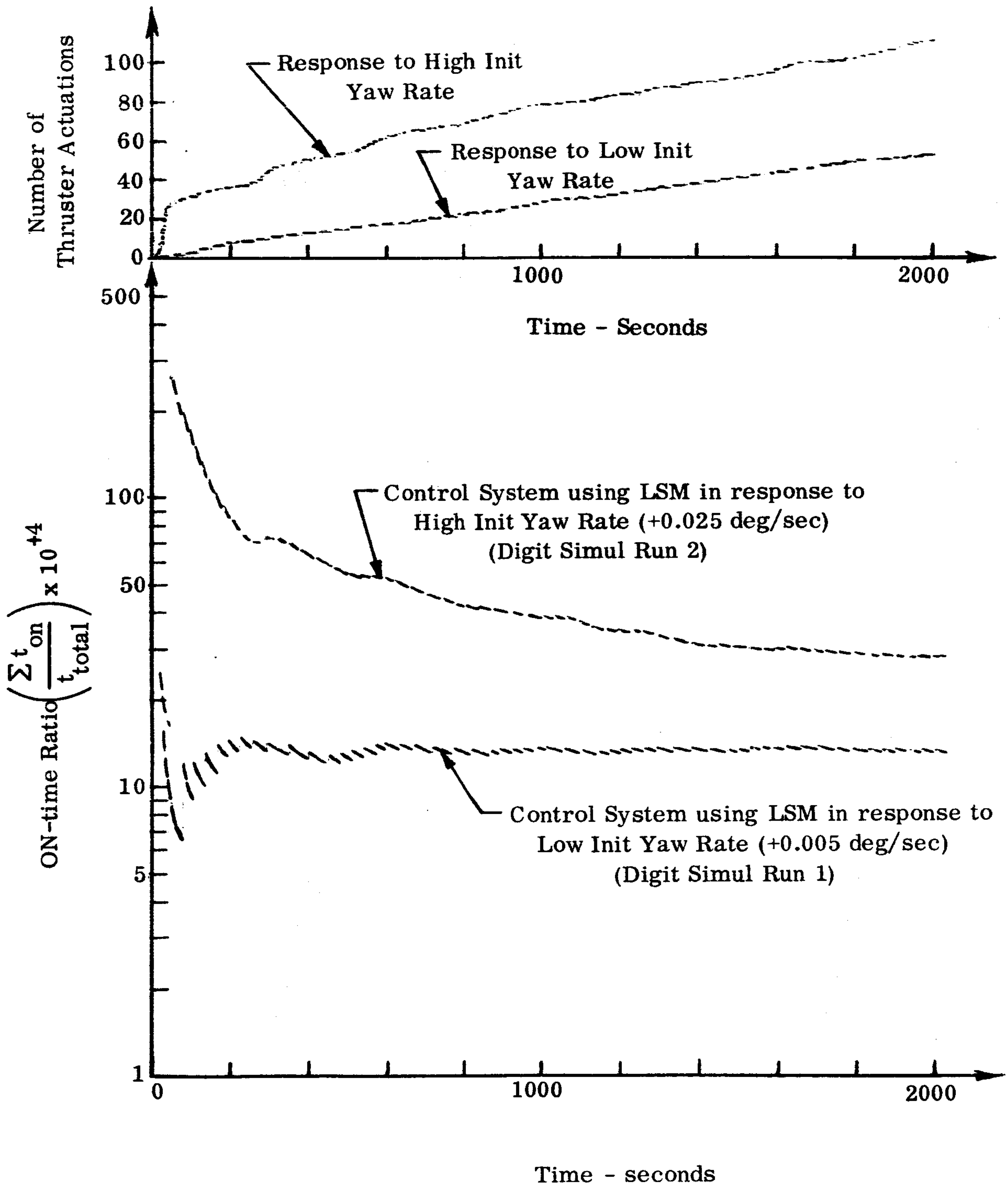

Figure 28. Influence of Initial Yaw Rate on LSM ON-Time Ratio and Number of Thruster Actions (No Disturbance)

which is indicative of "smoothness" of operation, possible chattering during acquisition, and the general demands on the thrusters. The other operating characteristic recorded in the simulation runs and shown in the Figure 28 is the ON-time ratio, which is defined here as the ratio of the total firing time of all thrusters to the total operating time:

$$\text{ON-time ratio} = \frac{\sum t_{on}}{t_{total}} \sim \frac{\text{seconds}}{\text{seconds}}$$

This operating characteristic is indicative of the system's overall propellant comsumption.

From the curves of Figure 28, it can be seen that the steady state propellant consumption of the dual axes LSM control system differs by a factor of about 2:1 as a result of a small variation in initial yaw rate from 0.005 degree/second to 0.025 degree/second. Digital simulation runs 3 and 4 (as listed in Figure 27) show a similar dependency on the initial conditions.

Since the LSM system does not include means for assuring that steady state operation will necessarily be the "best" (low average frequency) limit cycle mode, a conservative evaluation of a LSM system should assume the existence of worst case limit cycling which occurs during limit cycling with the yaw attitude periodically being close to the allowable control threshold, as discussed in Section 3.0. In all the following digital simulation disturbance runs, with LSM and DOTC systems, the following set of initial conditions was chosen as representative of such a worst case. The selected set of initial conditions was the following:

Roll: $\dot{\phi}_x(O) = +0.025$ deg/sec. $\phi_x(O) = -0.125$ deg.
Yaw: $\dot{\phi}_z(O) - +0.001$ deg/sec. $\phi_z(O) = +0.94$ deg.

The initial condition attitude angle values were chosen such that yaw was close to and approaching its allowable threshold value ($\phi_{DB} = \pm 1$ deg.) and such that during the first moments of operation, no engine firing would be commanded with either the LSM or the DOTC system (i.e., the system is just within its threshold).

Simulation runs 5 and 6 show convergence characteristics from these initial conditions with no disturbances acting on the systems. Figure 29 shows the two evaluation variables plotted versus time for runs 5 and 6. The reduction in propellant consumption and thruster actuations that was achieved by the DOTC system is quite apparent; the DOTC system achieves a total engine ON-time per total operating time that is about 1/5 of that required by the system using LSM. The number of required thruster actuations is also significantly less for the DOTC system.

5.2.2 Comparative Evaluations of LSM and DOTC Systems Operating in the Presence of Roll and Yaw Disturbances

Simulation runs were made to compare the effect of disturbances on two dual axis control systems. One system employed LSM and the other employed the DOTC-DBB-DYT system.

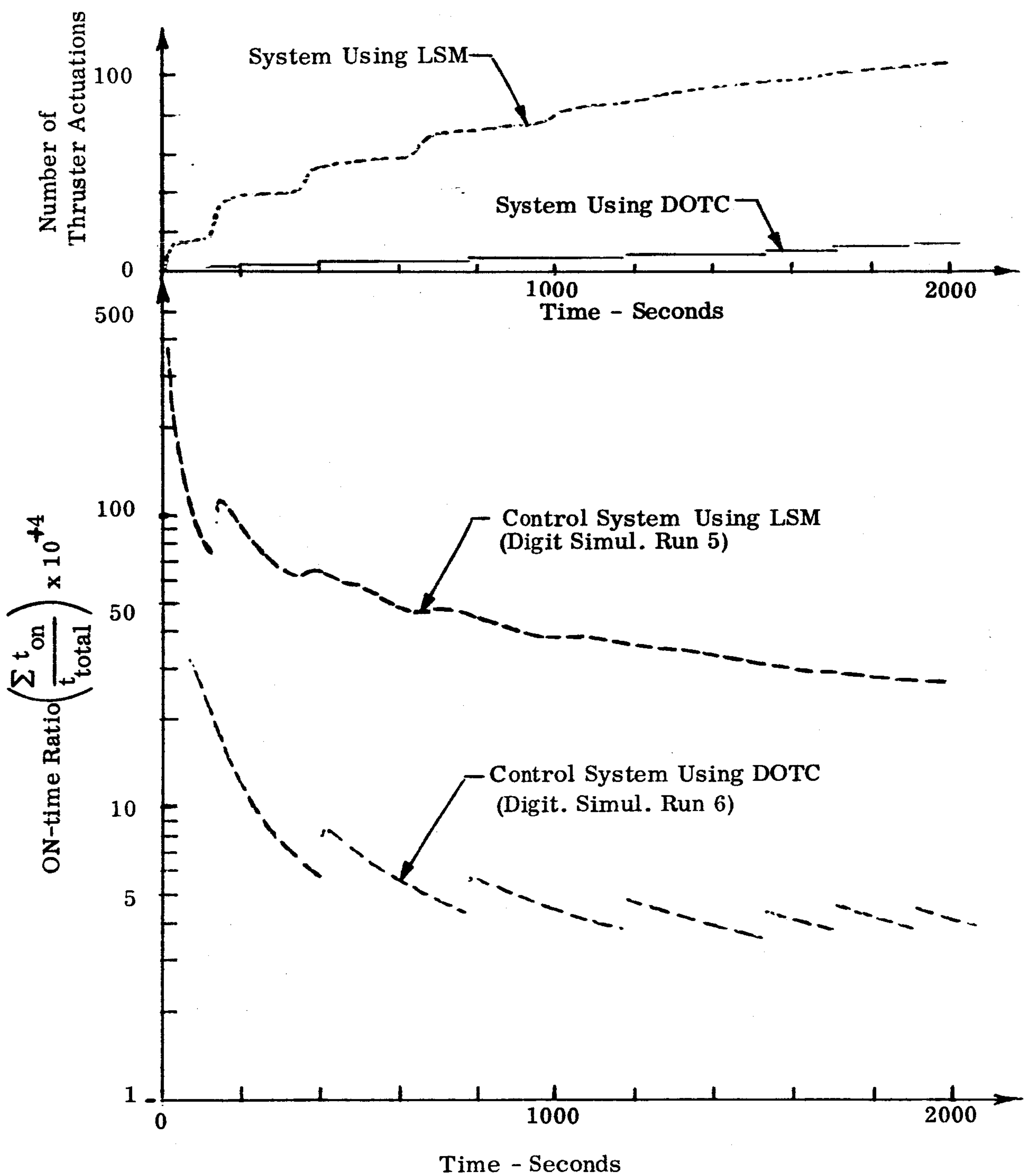

Figure 29. LSM and DOTC ON-Time Ratio and Thruster Actuation Time Histories for Representative Worst Case Initial Conditions (No Disturbance)

The previously discussed set of initial conditions was selected in order to present a fairly severe limit cycle condition for the LSM system.

The disturbances, type and magnitude, were chosen to be representative of the actual disturbances that would apply to the class of vehicle and the type of mission considered in this study.

Sets of comparative evaluation runs were made with identical starting conditions, identical general system characteristics, and identical disturbance conditions. The only difference between two simulation runs of a comparison set was that one system used LSM while the other system employed DOTC to control the firing of its attitude control thrusters.

The performances of the two systems under the various operating conditions were evaluated on the basis of the ON-time ratio, convergence and steady state propellant consumption characteristics, and the attitude control thruster actuation rate.

5.2.2.1 Effects of Small Roll Disturbances

Simulation runs were performed to investigate the effects of small roll disturbance effects due to possible pitch thruster misalignments and/or c. g. Shifts.

5.2.2.1.1 Transient Characteristics

Figure 30 and 31 present the two evaluation parameters, thruster ON-time ratio and number of thruster actuations, plotted as functions of time for both LSM and DOTC for the case of a constant roll acceleration equal to 50 x 10^{-6} degree/sec^2. Figure 30 indicates that the rate of convergence of the ON-time ratio is about the same for both systems. However, after about 1000 seconds of operation, the DOTC system is seen to have reached an ON-time ratio of one-sixth that of LSM with only about one-sixth the number of thruster actuations required by LSM.

Figure 31 shows the same parameters for a constant roll acceleration of 100 x 10^{-6} degrees per sec^2. The results are generally similar to those obtained with the smaller disturbance and the DOTC system converges to an ON-time ratio of about 30% of that of LSM with about 30% as many thruster actuations.

Comparison of the DOTC time histories of Figures 30 and 31 shows that doubling the roll disturbance level increases the ON-time ratio by about a factor of two. This increase in ON-time ratio with disturbance level is typical of a system that exhibits "one sided" limit cycling under the influence of the constant disturbance (i. e. the system oscillates about one side of its control threshold).

Comparison of the LSM time histories of Figure 30 and 31 indicates that, as a result of the larger LSM roll bit, the influence of the small disturbances on the performance of the system using LSM is only slight. The LSM system remains in a "two sided" limit cycle operation exhibiting a slight "wandering" limit cycle effect due to the roll disturbance.

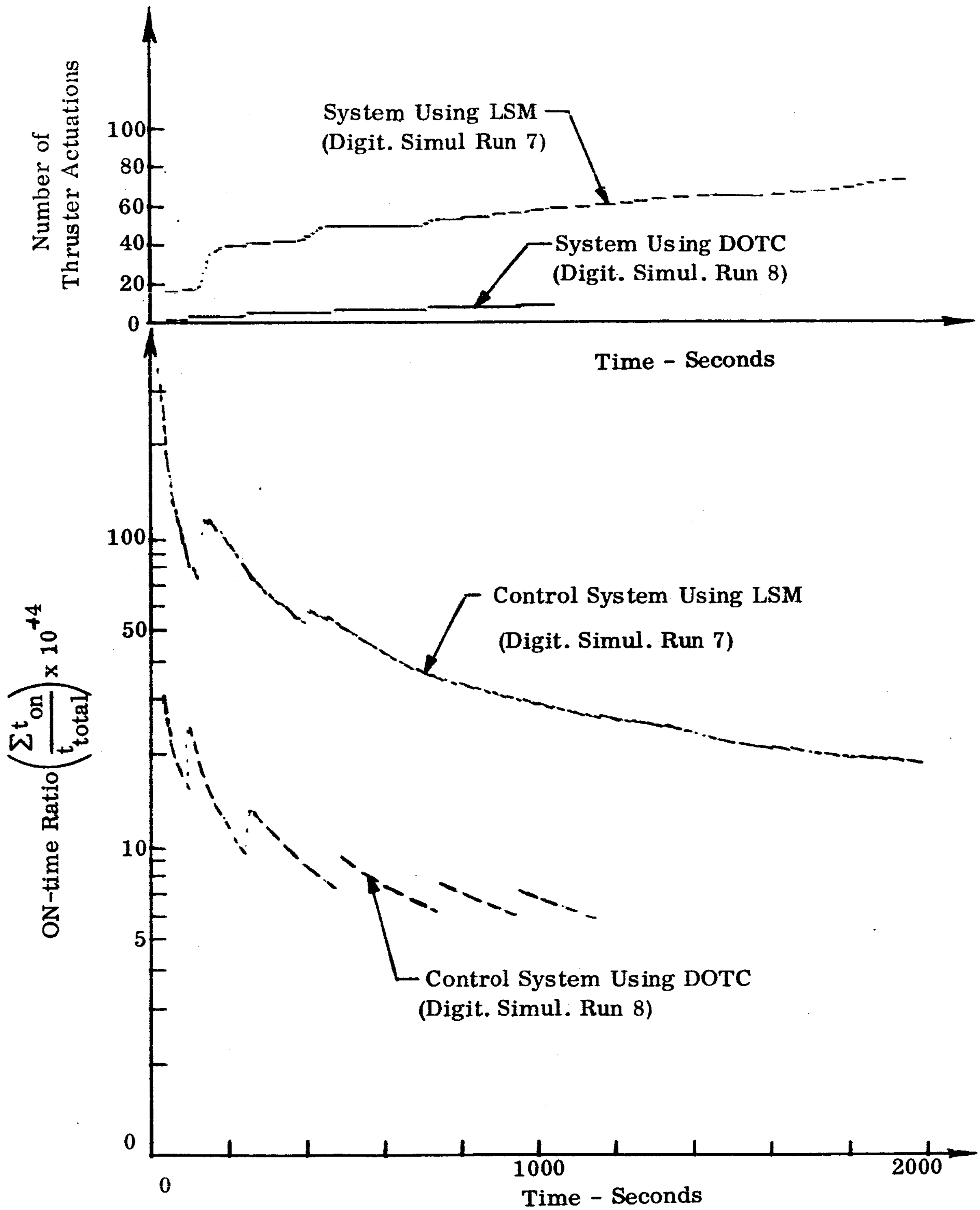

Figure 30. LSM and DOTC ON-Time Ratio and Thruster Actuation Time History (Roll Disturbance = 50×10^{-6} deg/sec^2)

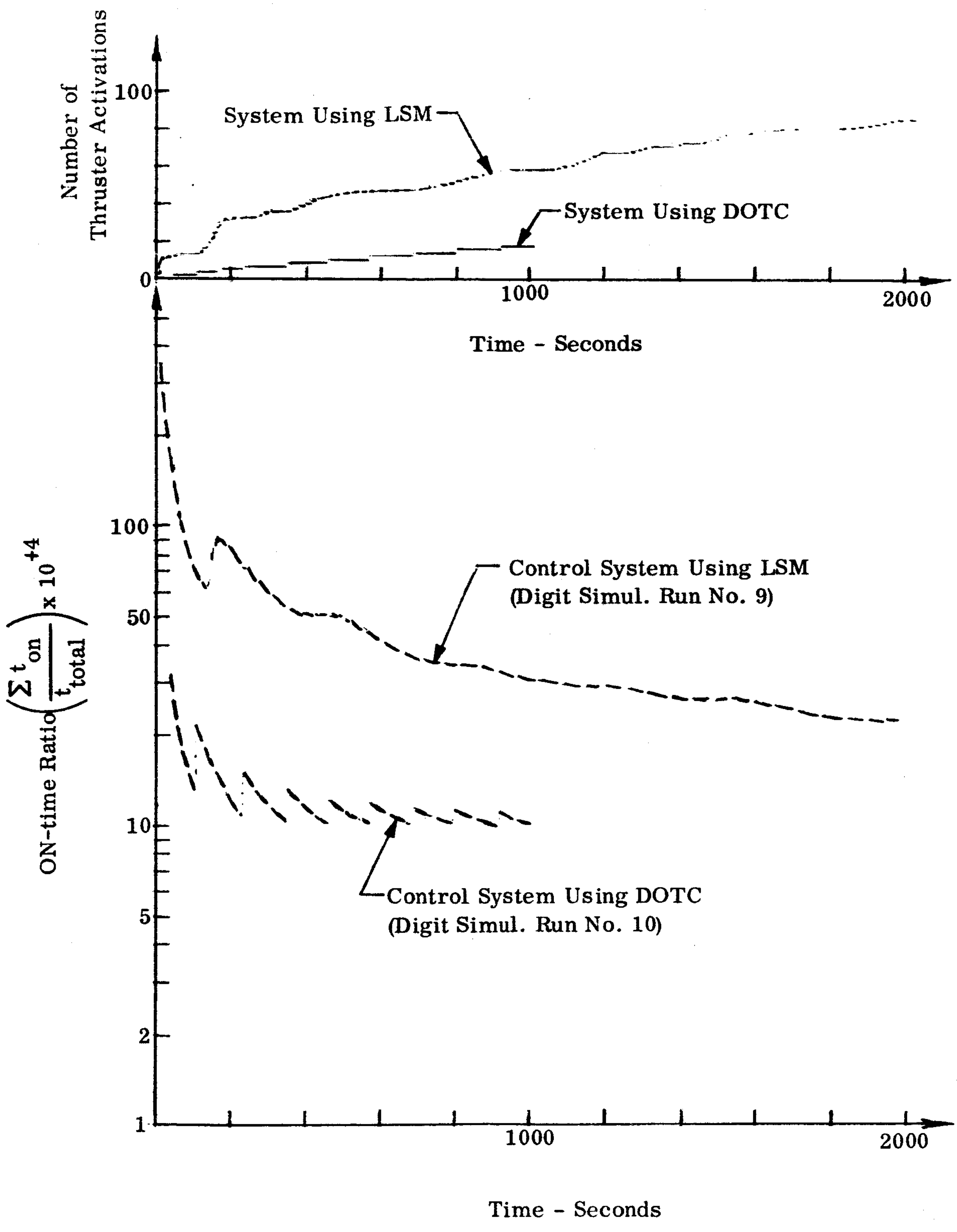

Figure 31. LSM and DOTC ON-Time Ratio and Thruster Actuation Time History (Roll Disturbance = 100×10^{-6} deg/sec^2)

Figure 32 summarizes the steady state values of ON-time ratio and thrusters firing rate for low level roll disturbance accelerations for LSM and DOTC.

5.2.2.2 Effects of Large Roll Disturbances

Digital runs 19 to 21 (See Figure 27) were performed in order to compare the LSM and DOTC system performance in the presence of large sustained disturbances equal to 25% of the roll control acceleration per engine. The ON-time ratio of the DOTC system, because of the small bits provided by the back to back mode was about 3 times that of LSM as shown by comparison of runs 19 and 20 of Figure 27. Run 21 was performed in order to demonstrate the effectiveness of the back to back mode disabling signal, "Q", discussed in Section 3.0. For this run, the back to back firing mode was disabled and the relative ON-time of LSM and DOTC were about the same value, 0.25. These runs demonstrate that, in the event of a command or a sustained large disturbance, such as might be produced by firing a ΔV rocket, the DBB mode of the combined DOTC logic should be disabled and that the "Q" signal can accomplish this effectively.

5.2.2.3 Effect of Yaw Disturbances

The effects of small yaw disturbances were evaluated by means of runs 11 through 15. The DOTC system yielded slightly better performance than LSM as a result of the differential yaw shutoff feature of the DOTC-DBB-DYT concept.

The DOTC system and LSM system are both operating in a one-sided limit cycle on the positive yaw threshold as a result of the positive yaw disturbance. In this mode of steady state operation the ON-time ratio becomes equal to the ratio of disturbance acceleration to control power, as may be seen by reference to Figure 27 (runs 11 through 15). Also, since the limit cycling is one-sided, the effects of a constant disturbance of the same gain magnitude are similar. This phenomenon is illustrated by the results of runs 11 and 15.

The influence of low magnitude yaw disturbances on the steady state performance of the DOTC system and the LSM system is illustrated by the curves of Figure 33. For these curves it is seen that the performance obtainable from DOTC is better than or equal to that obtainable from LSM. For the larger magnitudes of yaw disturbance the two control systems considered here will exhibit almost identical limit cycle performance. The LSM system will command firing and terminate firing of the two yaw control engines at slightly different times thus obtaining roll control action; the DOTC system will start the two engines simultaneously but will terminate the yaw control action in a "Differential Shut Off" fashion to obtain the required Roll control action. In general the roll control obtainable with DOTC will be tighter than that achievable with LSM.

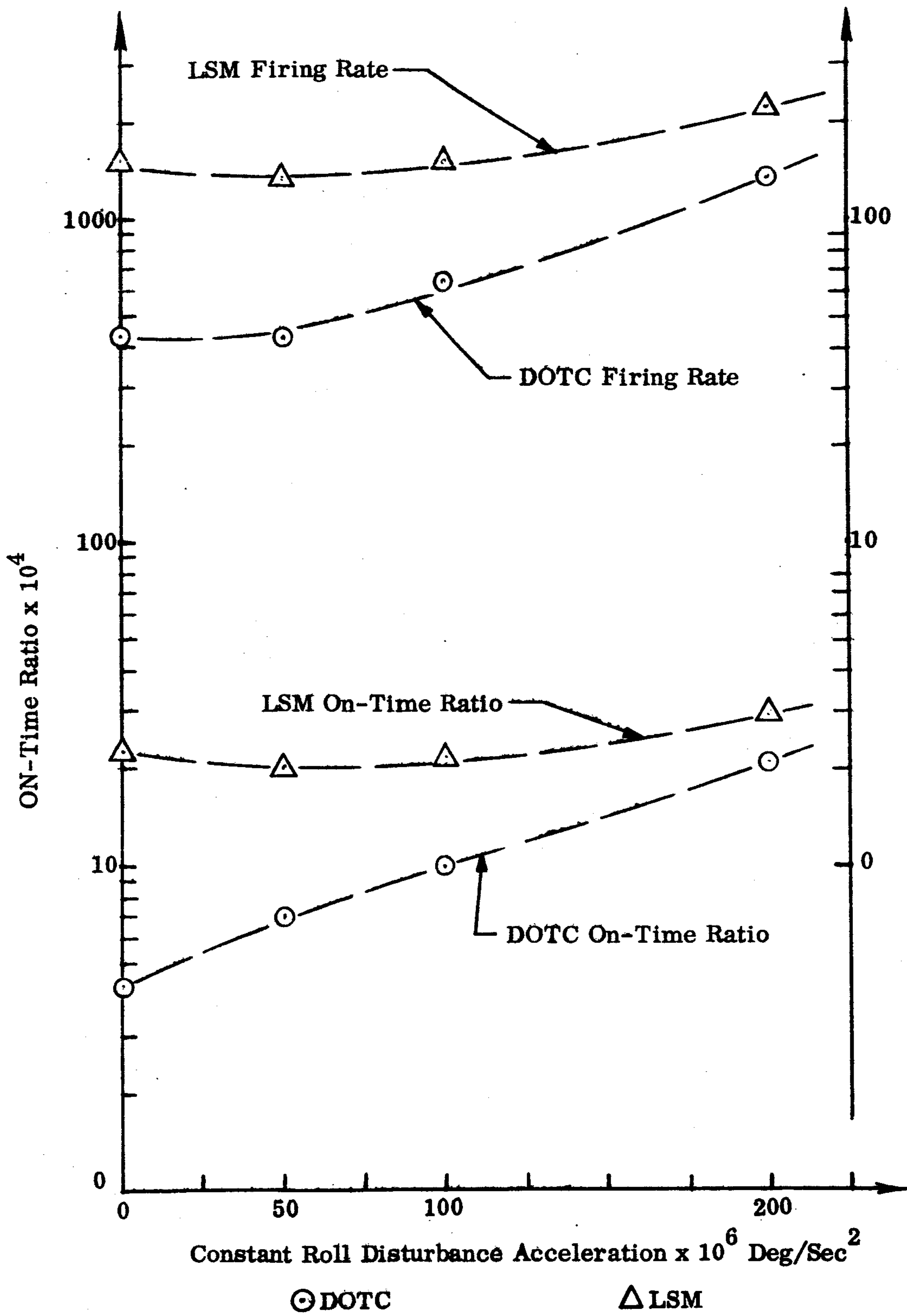

Figure 32. LSM and DOTC ON-Time Ratio and Thruster Firing Rate versus Roll Disturbance Level

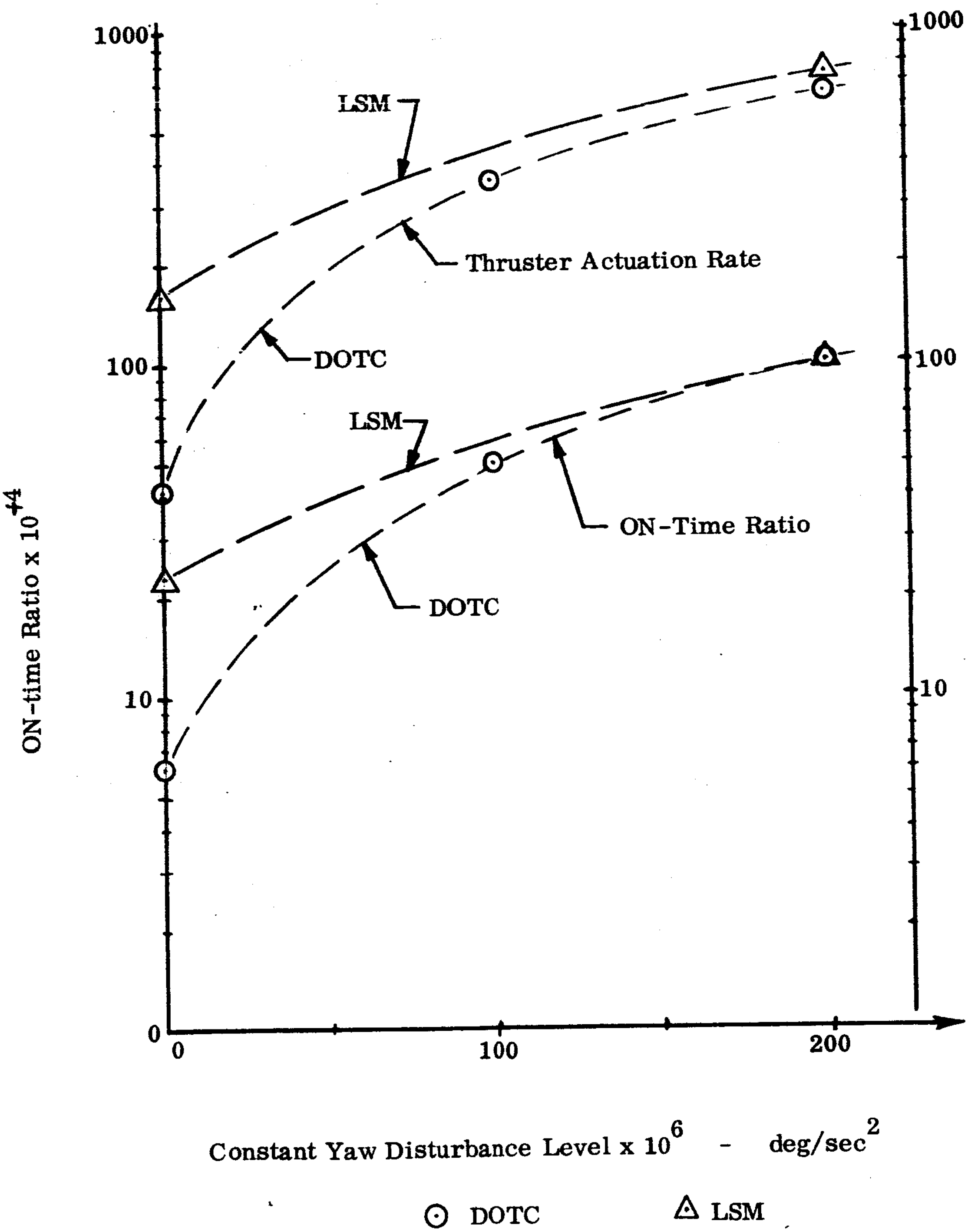

Figure 33. LSM and DOTC ON-Time Ratio and Thruster Firing Rate versus Yaw Disturbance Level

6.0 SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

6.1 SUMMARY AND CONCLUSIONS

This study has shown Differential On-Time Control to be a potentially important attitude control concept. DOTC yields significant performance advantages over the other systems investigated during undisturbed limit cycle operation, and at least comparable performance in disturbance situations. Moreover, an inherent ability to provide impulse bits of any size or degree of asymmetry (above a small practical limit) gives DOTC a growth potential to achieve nearly optimal performance in both disturbed and undisturbed situations. This growth potential was demonstrated but not pursued in this program.

The fundamental factor underlying the performance advantages of DOTC is its unique ability to provide a net impulse bit, Δt, which is much smaller than the minimum impulse bit of a single thruster, t_{min}. The ratio of the smallest achievable Δt to t_{min} is the most important parameter affecting the performance of DOTC relative to other systems.

The principal disadvantages of DOTC are related to its hardware implementation. Approximately 100 digital logic gates and flip-flops would be required for the complete DOTC-DBB-DYT system. Rate sensor accuracy demands are directly proportional to the propellant savings, because these savings are achieved primarily by reducing the limit cycle rates. Greater demands are placed on the thruster system also. The threshold-type DOTC logic described in this report is insensitive to all but the turn-off delay of the thrusters, but this must be small, repeatable, and consistent from thruster to thruster if the necessary small Δt's are to be achieved. Most of the comparisons and conclusions in this report are based on a Δt of 0.01 seconds and a $\Delta t/t_{min}$ ratio of 0.2, although the effect of variations in these parameters are also shown. These values are within the present state-of-the-art.

The following specific conclusions can be drawn for the baseline vehicle. Undisturbed limit cycle operation is assumed unless otherwise stated.

(1) In the single axis (roll) case, DOTC-DBB shows an advantage over a conventional system in terms of rate of propellant consumption for $\Delta t/t_{min} < 0.414$, and in terms of thruster actuation rate for $\Delta t/t_{min} < 0.5$.

(2) In the dual axis case with $\Delta t/t_{min} = 0.2$, DOTC-DBB-DYT reduces both the propellant consumption and thruster actuation rates to approximately 50% of those achieved by Linear Signal Mixing operating in its most favorable mode. These best LSM rates, in turn, are only about 25% of those required by a conventional ON-OFF system. (Worst case LSM performance is roughly comparable to that of a conventional system.) Hence, by utilizing DOTC, the propellant consumption and thruster actuation rates are both reduced to about

12% of those for a conventional system (or LSM operating in an unfavorable mode). In terms of propellant mass, the savings compared to the LSM best case is approximately 9 kg per day. This minimum daily saving of propellant is sufficient to impart a roll rate of about 30 deg/sec to the baseline vehicle.

(3) The Differential Back-to-Back mode of DOTC is more efficient than the Differential Yaw Thrusting mode, in their simplest forms, and during undisturbed operation. A combined DOTC-DBB-DYT system is required, however, if reasonable performance in disturbance and large transient situations as well as good undisturbed performance is to be provided.

(4) In both disturbed and undisturbed operation, DOTC-DBB-DYT performance is more predictable than LSM. DOTC converges to its steady state values of propellant consumption and thruster actuation rates much more quickly than LSM. In addition, DOTC steady state performance is virtually independent of initial conditions of attitudes and rates, whereas relatively small changes in initial conditions spell the difference between the best and worst cases of LSM performance.

(5) DOTC-DBB-DYT performance in the presence of roll and yaw disturbances between zero and the maximum values encountered in a 184 km (100 n. mi.) orbit is as good as or better than LSM, with the largest advantage occurring at the lowest disturbances. In this comparison, initial conditions were used which correspond to a fairly severe mode of LSM operation in the undisturbed situation; however, it is believed that LSM performance under disturbed conditions, is substantially less dependent upon initial conditions. In the presence of large roll disturbances, such as may occur during main engine firings, DOTC-DBB-DYT performance is three times worse than LSM — unless the DBB mode is disabled, in which case comparable performance is obtained. The DBB mode can be readily disabled in that situation.

6.2 RECOMMENDATIONS

The study program described in this report was undertaken to evaluate the feasibility and potential usefulness of DOTC logic concepts. Such feasibility was shown by studying variations of one basic approach - using threshold logic with fixed system parameters - and by developing a system capable of superior undisturbed performance and comparable disturbed performance. Because of the potential importance shown for this type of system it is recommended the DOTC concept be further pursued in two directions:

1. Other basic approaches to DOTC, several of which were touched on but not exploited in this study, should be investigated to the same general depth as the threshold approach has been or until they can be ruled out.

2. The existing DOTC approach (or an even more promising alternate, if one is found) should be further developed, to a point where a decision can be made regarding the desirability of physically implementing it. The following specific recommendations are made for accomplishing these objectives.

(a) The statistical variations in the differential on-time which will be encountered as extremely small values of Δ t are commanded, and the effects of these statistical variations on system performance, should be investigated because of the key importance of this parameter to the DOTC concept.

(b) The feasibility of several alternate approaches to DOTC should be investigated. Promising alternatives include concepts based on using variable rather than fixed thresholds to achieve optimal impulse bit sizes as a function of disturbance level; concepts which intentionally develop asymmetrical limit cycles to achieve very low rates in one direction; concepts based on using computer, rather than threshold, control of the differential ON-time to take advantage of past performance history as well as the current situation; and concepts based on combining LSM, which has an excellent ability to select the proper thruster to fire in any situation, with DOTC to achieve the additional advantage of small (or variable) impulse bits.

(c) The existing DOTC concept should be further evaluated and developed. First, the influence of small disturbances on both DOTC and LSM should be studied in more detail than was possible in the current program. Analytical methods of estimating disturbed performance of both systems should be investigated. Second, the overall performance of a DOTC system should be evaluated for several complete missions, to determine the relative overall importance of undisturbed, disturbed, and transient performance. It is desirable to know, for example, what sort of missions involve enough low disturbance operation that the propellant savings realized from DOTC are a significant part of the entire attitude control propellant requirement. Third, the design of a practical DOTC system should be carried out in enough detail to evaluate its reliability, cost, size, and weight compared to other approaches so that these may be traded off against the improved performance and a decision may be made regarding the desirability of designing, building, and testing a prototype DOTC system.

7.0 REFERENCES

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